Efficiency of Liability Rules: Some Further Results

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Abstract

The main purpose of this paper is to show that the set of efficient rules which apportion liability between the victim and the tortfeasor is much larger than is generally believed to be the case. A larger set of efficient rules in general would have the implication of a less sharp conflict between economic efficiency on the one hand and non-efficiency normative criteria, including fairness and restitutive justice, on the other. The condition of negligence liability which characterizes efficiency in the context of liability rules has an all-or-none character. Negligence liability requires that if one party is negligent and the other is not then the liability for the entire accident loss must fall on the negligent party. Thus within the framework of standard liability rules efficiency requirements preclude any non-efficiency considerations in situations where one party is negligent and the other is not. In this paper it is shown that a part of accident loss plays no part in providing appropriate incentives to the parties for taking due care and can therefore be apportioned on non-efficiency considerations without in any way compromising the social goal of efficiency. For a systematic analysis of the requirements for efficiency, in this paper a notion more general than that of a liability rule, namely that of a decomposed liability rule, is introduced. A complete characterization of efficient decomposed liability rules is provided in the paper. The most important implication of the theorems of this paper is that by decomposing accident loss in two parts, the scope for non-efficiency considerations can be significantly broadened without sacrificing economic efficiency.

Key Words: Tort Law, Liability Rules, Decomposed liability Rules, Efficient Rules, Nash Equilibria, Negligence Liability, Restitutive Justice
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Considerations relating to the efficiency of liability rules have occupied an important place in the law and economics literature right from its inception. The pioneering contribution by Calabresi (1961) analyzed the effect of liability rules on parties' behaviour. In his seminal contribution Coase (1960) looked at liability rules from the point of view of their implications for social costs. The rule of negligence was analyzed by Posner (1972) from the perspective of economic efficiency. The first formal analysis of liability rules was done by Brown (1973). His main results demonstrated the efficiency of both the rule of negligence and the rule of strict liability with the defense of contributory negligence. Formal treatment of some of the most important results of the extensive literature on liability rules is contained in Landes and Posner (1987), Shavell (1987), and Miceli (1997). A complete characterization of efficient liability rules is contained in Jain and Singh (2002).

In the literature dealing with the question of efficiency of liability rules, the problem has generally been considered within the framework of accidents resulting from interaction of two risk-neutral parties, the victim and the injurer. The social goal is taken to be the minimization of total social costs, which are defined to be the sum of costs of care taken by the two parties and expected accident loss. The probability of accident and the amount of loss in case of occurrence of accident are assumed to depend on the levels of care taken by the two parties. A party is called nonnegligent if its care level is at least equal to the due care level; otherwise it is called negligent. A liability rule determines the proportions in which the two parties are to bear the loss in case of occurrence of accident on the basis of whether and by what proportions the parties involved in the interaction are negligent. A liability rule is efficient if it invariably induces both parties to behave in ways which result in socially optimal outcomes, i.e., outcomes under which total social costs are minimized. The central result regarding the efficiency question that has emerged is that a liability rule is efficient if and only if it satisfies the condition of negligence liability. The condition of negligence liability requires that (i) if the victim is nonnegligent and the injurer is negligent then the entire loss, in case of occurrence of accident, must be borne by the injurer; and (ii) if the injurer is nonnegligent and the victim is negligent then the entire loss, in case of occurrence of accident, must be borne by the victim.

The condition of negligence liability, which has an all-or-none character, completely specifies the assignment of liability shares in cases where one party is negligent and the other is not. Consequently, it would seem that if the choice of a liability rule is to be from the set of efficient liability rules then the non-efficiency considerations, including those of fairness and restitutive justice, cannot possibly play any role in assigning liability in cases where one party is negligent and the other is not. Other considerations at best can have a role only in situations when either both parties are negligent or both are nonnegligent.

If liabilities of the parties are to be specified as proportions of total accident loss, as is done in tort law, then what has been said above about efficiency considerations precluding other considerations in cases where one party is negligent and the other nonnegligent is indeed correct. But if one is willing to go outside the traditional tort law framework then it turns out that the scope for non-efficiency considerations is much greater than is generally thought to be the case. In providing correct incentives to the parties, part of accident loss, equal to the optimal loss when both parties are taking the due care, suitably adjusted to take into account differing probabilities of accident with different care levels, seems to play no role and can therefore be apportioned between the two parties independently of their care levels. It is the apportionment of the accident loss over and above the adjusted optimal loss which turns out to be crucial from the point of view of providing correct incentives to the parties. An example may help illustrate the point.

Consider a two-party interaction in which the accident occurs with certainty; but the magnitude of accident loss depends on the care levels of the parties. Let the loss be 100 if neither party takes care, 98.5 if one party takes care and the other does not, and 97 if both parties take care. Let taking care by either party cost 1. As the rule of strict liability with the defense of contributory negligence is an efficient liability rule, it would induce in the context of the scenario of this example both the parties to take care and thus lead to the socially optimal outcome. It can easily be checked that the rule would lead to the socially optimal outcome of both parties taking care even if part of the loss equal to the optimal loss, which is 97 here, is assigned to the injurer regardless of the care levels of the two parties.

This example makes it clear that the efficiency requirement does not preclude altogether a role for non-efficiency considerations even when one party is negligent and the other is not. In principle, part of the accident loss can be assigned between the parties purely on non-efficiency considerations without affecting the efficiency property. For a systematic treatment of this question the notion of a liability rule needs to be generalized so that all possible decompositions of accident loss could be considered to find out the precise constraints imposed by the efficiency requirement.

A liability rule apportions the accident loss between the parties as a function of the parties' proportions of nonnegligence. Corresponding to any liability rule one can define a two-parameter family of rules in the following way: (i) A specified multiple (θ) of adjusted optimal loss is to be assigned between the two parties in fixed proportions $(\lambda, 1-\lambda)$ (ii) The remainder of the loss is to be apportioned between the two parties as specified by the liability rule in question as a function of proportions of nonnegligence of the parties. This more general notion of a rule would be called a (λ, θ)-decomposed liability rule. If $\theta = 0$ then the notion of a decomposed liability rule coincides with that of a liability rule. We show that if $0 \leq \theta \leq 1$ then a decomposed liability rule is efficient if and only if the corresponding liability rule is efficient. If $\theta > 1$ then it turns out that no decomposed liability rule can be efficient. In other words, efficiency properties remain unaffected provided the quantum of loss that is assigned independently of care levels does not exceed adjusted optimal loss. Regardless of whether a decomposed liability rule corresponds to an efficient liability rule or an inefficient liability rule, if $\theta > 1$ then the decomposed liability rule would be inefficient. In view of these results it is clear that it is the amount of loss that is in excess of the adjusted optimal loss which constitutes the irreducible minimum which must be assigned to the negligent party to ensure efficiency, in cases where one party is negligent and the other is not. Thus the requirements imposed by efficiency considerations could be quite mild depending on the context.

The paper is divided in three sections. The first section sets out the framework of analysis and introduces the notion of a decomposed liability rule. The next section states and discusses the main results: (i) Given that $0 \leq \theta \leq 1$, a (λ, θ)-decomposed liability rule is efficient if and only if it satisfies the condition of negligence liability; and (ii) For $\theta > 1$ every (λ, θ)-decomposed liability rule is inefficient. All proofs are given in the appendix at the end of the paper. The last section of the paper discusses the significance of the results.

1 Definitions and Assumptions

We consider accidents resulting from interaction of two parties, assumed to be strangers to each other, in which, to begin with, the entire loss falls on one party to be called the victim (plaintiff). The other party would be referred to as the injurer (defendant). We denote by $a \geq 0$ the index of the level of care taken by the victim; and by $b \geq 0$ the index of the level of care taken by the injurer.

Let

$A = \{a \mid a \geq 0$ is the index of some feasible level of care which can be taken by the victim$\}$, and

$B = \{b \mid b \geq 0$ is the index of some feasible level of care which can be taken by the injurer$\}$.

We assume:

$0 \in A \ \wedge \ 0 \in B.$ (A1)

We denote by $c(a)$ the cost to the victim of care level a and by $d(b)$ the cost to the injurer of care level b.

Let $C = \{c(a) \mid a \in A\}$, and $D = \{d(b) \mid b \in B\}$.
We assume:

$$c(0) = 0 \ \wedge \ d(0) = 0. \tag{A2}$$

We also assume that c and d are strictly increasing functions of a and b respectively.

(A3)

In view of (A2) and (A3) it follows that:
$(\forall c \in C)(c \geq 0) \ \wedge \ (\forall d \in D)(d \geq 0)$.
A consequence of (A3) is that c and d themselves can be taken as indices of levels of care of the two parties.

Let Π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both Π and H will be assumed to be functions of c and d; $\Pi = \Pi(c,d), H = H(c,d)$. Let $L = \Pi H$. L is thus the expected loss due to accident.
We assume:

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ L(c,d) \leq L(c',d)] \ \wedge \ [d > d' \ \rightarrow \ L(c,d) \leq L(c,d')]]. \tag{A4}$$

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss.

Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and the expected loss due to accident; $TSC = c + d + L(c,d)$. Let $M = \{(c',d') \in C \times D \mid c' + d' + L(c',d')$ is minimum of $\{c + d + L(c,d) \mid c \in C \ \wedge \ d \in D\}\}$. Thus M is the set of all costs of care configurations (c',d') which are total social cost minimizing. It will be assumed that:

C, D and L are such that M is nonempty. (A5)

Let I denote the closed unit interval$[0,1]$[1]. Given C, D, Π, H and $(c^*, d^*) \in M$, we define functions p and q as follows:

$p : C \mapsto I$ by:

$$\begin{aligned} p(c) &= 1 \quad \text{if } c \geq c^* \\ &= \tfrac{c}{c^*} \quad \text{if } c < c^*; \end{aligned}$$

$q : D \mapsto I$ by:

$$\begin{aligned} q(d) &= 1 \quad \text{if } d \geq d^* \\ &= \tfrac{d}{d^*} \quad \text{if } d < d^*. \end{aligned}$$

[1]In addition to denoting the set $\{x \mid 0 \leq x \leq 1\}$ by $[0,1]$, we use the following standard notation to denote:
by $[0,1)$ the set $\{x \mid 0 \leq x < 1\}$,
by $(0,1]$ the set $\{x \mid 0 < x \leq 1\}$, and
by $(0,1)$ the set $\{x \mid 0 < x < 1\}$.

In case there is a legally binding due care level for the plaintiff, it would be taken to be identical with c^* figuring in the definition of function p; and in case there is a legally binding due care level for the defendant, it would be taken to be identical with d^* figuring in the definition of function q. Thus implicitly it is being assumed that the legally binding due care levels are always set appropriately from the point of view of minimizing total social costs.

p and q would be interpreted as proportions of nonnegligence of the victim and the injurer respectively. $(1-p)$ and $(1-q)$ consequently would denote the proportions of negligence of the victim and the injurer respectively.

Let $\lambda \in [0,1]$ and $\theta \geq 0$. Denote $H(c^*, d^*), \Pi(c^*, d^*)$ and $L(c^*, d^*)$ by H^*, Π^* and L^* respectively.

Define functions S and G as follows:

$$S(c,d) = H(c,d) - \theta H^* \tfrac{\Pi^*}{\Pi(c,d)} \quad \text{if } \Pi(c,d) \neq 0 \ \wedge \ H(c,d) > \theta H^* \tfrac{\Pi^*}{\Pi(c,d)}$$
$$= 0 \quad \text{otherwise.}$$

$$G(c,d) = \theta H^* \tfrac{\Pi^*}{\Pi(c,d)} \quad \text{if } S(c,d) > 0$$
$$= H(c,d) \quad \text{if } S(c,d) = 0.$$

Let $R(c,d) = \Pi(c,d)S(c,d)$ and $F(c,d) = \Pi(c,d)G(c,d)$.
In other words,

$$R(c,d) = L(c,d) - \theta L^* \quad \text{if } L(c,d) > \theta L^*$$
$$= 0 \quad \text{otherwise.}$$

$$F(c,d) = \theta L^* \quad \text{if } L(c,d) > \theta L^*$$
$$= L(c,d) \quad \text{if } L(c,d) \leq \theta L^*.$$

Thus,

$$R(c,d) = max\{L(c,d) - \theta L^*,\ 0\}$$
$$F(c,d) = min\{L(c,d),\ \theta L^*\}.$$

$G(c,d)$ will be referred to as the specified multiple of the adjusted optimal loss; and $S(c,d)$ will be called the excess loss. Consistent with this nomenclature, $F(c,d)$ will be referred to as the specified multiple of the expected optimal loss; and $R(c,d)$ as the expected excess loss.

A liability rule is a rule which specifies the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of proportions of two parties' nonnegligence. Formally, a liability rule is a function f from I^2 to I^2, $f : I^2 \mapsto I^2$, such that: $f(p,q) = (x,y)$, where $x + y = 1$.

A (λ, θ) - decomposed liability rule is a rule which specifies the proportions in which the two parties are to bear the excess loss in case of occurrence of accident as a function of proportions of two parties' nonnegligence; and assigns specified multiple of adjusted optimal loss in the $(\lambda, 1-\lambda)$ proportions to the victim and the injurer respectively.

Formally, a (λ, θ) - decomposed liability rule is a function f from I^2 to I^2, $f : I^2 \mapsto I^2$, such that: $f(p,q) = (x,y) = [x(p,q), y(p,q)]$, where $x + y = 1$.

Let C, D, Π, H and $(c^*, d^*) \in M$ be given. If accident takes place and loss of $H(c,d)$ materializes, then $xS(c,d) + \lambda G(c,d)$ will be borne by the victim and $yS(c,d) + (1-\lambda)G(c,d)$ by the injurer. As, to begin with, in case of occurrence of accident, the entire loss falls upon the victim, $yS(c,d) + (1-\lambda)G(c,d)$ represents the liability payment by the injurer to the victim. The expected costs of the victim and the injurer, to be denoted by EC_1 and EC_2 respectively, therefore, are $[c + xR(c,d) + \lambda F(c,d)]$ and $[d + yR(c,d) + (1-\lambda)F(c,d)]$ respectively. Both parties are assumed to prefer smaller expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

Remark 1 *The notion of a (λ, θ) - decomposed liability rule is a generalization of the notion of a liability rule. For $\theta = 0$, the definition of a (λ, θ) - decomposed liability rule reduces to that of a liability rule.*

It should be noted that for every liability rule there corresponds a class of (λ, θ) - decomposed liability rules. The class corresponding to liability rule f is given by: $\{g \mid g$ is a (λ, θ) - decomposed liability rule, $\lambda \in [0,1], \theta \geq 0, (\forall p, q \in [0,1])[g(p,q) = f(p,q)]\}$.

Let f be a (λ, θ) - decomposed liability rule. An application of f consists of specification of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5). f is defined to be efficient for a given application C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5) iff $(\forall (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ and $(\exists (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium].[2] f is defined to be efficient iff it is efficient for every possible application. In other words, a (λ, θ) - decomposed liability rule is efficient for given C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5) iff (i) every Nash equilibrium is total social cost minimizing, and (ii) there exists at least one Nash equilibrium. A (λ, θ) - decomposed liability rule is efficient iff it is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5).

Remark 2 *It should be noted that if (A5) is not satisfied then no (λ, θ) - decomposed liability rule can be efficient.*

To illustrate some of the above ideas we consider below several examples.

Example 1: Let $(\frac{1}{3}, 1)$ - decomposed liability rule f be defined by:
$(\forall p \in [0,1])(\forall q \in [0,1])[f(p,q) = (\frac{1}{2}, \frac{1}{2})]$.
Consider an application of f such that:
$C = D = \{0, 1\}$;
$L(0,0) = 9, L(0,1) = L(1,0) = 7.5, L(1,1) = 6$.

[2] Throughout this paper we consider only pure-strategy Nash equilibria.

Here $(1,1)$ is the unique TSC-minimizing configuration of costs of care.
Let$(c^*, d^*) = (1,1)$.
As $\theta L^* = 6$, we obtain:
$R(0,0) = 3, R(0,1) = R(1,0) = 1.5, R(1,1) = 0$
$F(0,0) = F(0,1) = F(1,0) = F(1,1) = 6$.
$EC_1(0,0) = 3.5, EC_1(0,1) = 2.75, EC_1(1,0) = 3.75, EC_1(1,1) = 3$;
$EC_2(0,0) = 5.5, EC_2(0,1) = 5.75, EC_2(1,0) = 4.75, EC_2(1,1) = 5$.
Therefore $(0,0)$ is the only Nash equilibrium. Thus f is inefficient.

Example 2: Let $(\frac{1}{2}, \frac{1}{2})$ - decomposed liability rule f be defined by:
$(\forall p \in [0,1))(\forall q \in [0,1))[f(p,q) = (\frac{1}{2}, \frac{1}{2})] \wedge (\forall p \in [0,1))[f(p,1) = (1,0)] \wedge (\forall q \in [0,1))[f(1,q) = (0,1)] \wedge [f(1,1) = (\frac{1}{2}, \frac{1}{2}]$.
Consider an application of f such that:
$C = D = \{0,1\}$;
$L(0,0) = 10, L(0,1) = L(1,0) = 8.5, L(1,1) = 7$.
Let$(c^*, d^*) = (1,1)$, which is the unique TSC-minimizing configuration of costs of care.
As $\theta L^* = 3.5$, we obtain:
$R(0,0) = 6.5, R(0,1) = R(1,0) = 5, R(1,1) = 3.5$
$F(0,0) = F(0,1) = F(1,0) = F(1,1) = 3.5$
We have:
$EC_1(0,0) = 5, EC_1(0,1) = 6.75, EC_1(1,0) = 2.75, EC_1(1,1) = 4.5$
$EC_2(0,0) = 5, EC_2(0,1) = 2.75, EC_2(1,0) = 6.75, EC_2(1,1) = 4.5$
Here $(1,1)$ is the only Nash equilibrium. Thus the rule is efficient for the application considered here.[3]

Example 3: Let $(\frac{1}{2}, 2)$ - decomposed liability rule f be defined by:
$(\forall p \in [0,1))(\forall q \in [0,1))[f(p,q) = (\frac{1}{2}, \frac{1}{2})] \wedge (\forall p \in [0,1))[f(p,1) = (1,0)] \wedge (\forall q \in [0,1))[f(1,q) = (0,1)] \wedge [f(1,1) = (\frac{1}{2}, \frac{1}{2}]$.
Consider an application of f such that:
$C = D = \{0,1\}$;
$L(0,0) = 10, L(0,1) = L(1,0) = 8.5, L(1,1) = 7$.
Let $(c^*, d^*) = (1,1)$, which is the unique TSC-minimizing configuration of costs of care.
As $\theta L^* = 14$, it follows that:
$(\forall (c,d) \in C \times D)[R(c,d) = (0,0) \wedge F(c,d) = L(c,d)]$.
$EC_1(0,0) = 5, EC_1(0,1) = 4.25, EC_1(1,0) = 5.25, EC_1(1,1) = 4.5$

[3] $(\frac{1}{2}, \frac{1}{2})$ - decomposed liability rule of Example 2 is efficient for every C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5). Efficiency of this decomposed liability rule for every application follows from Theorem 1.

$EC_2(0,0) = 5, EC_2(0,1) = 5.25, EC_2(1,0) = 4.25, EC_2(1,1) = 4.5$
Consequently, $(0,0)$ is the only Nash equilibrium; and thus the rule is inefficient.

Example 4: Let f be a $(0,1)$ - decomposed liability rule defined by:
$(\forall p \in [0,1])(\forall q \in [0,1))[f(p,q) = (0,1)] \ \wedge \ (\forall p \in [0,1])[f(p,1) = (1,0)]$.
Consider:
$C = D = \{0,1,2\}$;
$L(0,0) = 10, L(0,1) = 6, L(0,2) = 5, L(1,0) = 5.5, L(1,1) = 1.5, L(1,2) = .5, L(2,0) = 5, L(2,1) = 1, L(2,2) = 0$.
Here $M = \{(1,1),(1,2)\}$. Let $(c^*, d^*) = (1,1)$. Hence, $L^* = 1.5$
Therefore,
$R(0,0) = 8.5, R(0,1) = 4.5, R(0,2) = 3.5, R(1,0) = 4, R(1,1) = 0, R(1,2) = 0, R(2,0) = 3.5, R(2,1) = 0, R(2,2) = 0$; and
$F(0,0) = F(0,1) = F(0,2) = F(1,0) = F(1,1) = F(2,0) = 1.5, F(1,2) = .5, F(2,1) = 1, F(2,2) = 0$.
We have:
$EC_1(0,0) = 0, EC_1(0,1) = 4.5, EC_1(0,2) = 3.5, EC_1(1,0) = 1, EC_1(1,1) = 1,$
$EC_1(1,2) = 1, EC_1(2,0) = 2, EC_1(2,1) = 2, EC_1(2,2) = 2;$
$EC_2(0,0) = 10, EC_2(0,1) = 2.5, EC_2(0,2) = 3.5, EC_2(1,0) = 5.5, EC_2(1,1) = 2.5,$
$EC_2(1,2) = 2.5, EC_2(2,0) = 5, EC_2(2,1) = 2, EC_2(2,2) = 2.$
Here there are two Nash equilibria, namely, $(1,1)$ and $(1,2)$.
The rule is efficient for the application considered here.[4]

Example 5: Let $(1,1)$ - decomposed liability rule f be defined by:
$(\forall p \in [0,1))(\forall q \in [0,1])[f(p,q) = (1,0)] \ \wedge \ (\forall q \in [0,1])[f(1,q) = (0,1)]$.
Let:
$C = \{0,1,2\}, D = \{0,1\}$;
$L(0,0) = 10, L(0,1) = 6, L(1,0) = 5, L(1,1) = 1, L(2,0) = 4, L(2,1) = 0$.
Here $M = \{(1,1),(2,1)\}$.
Let$(c^*, d^*) = (2,1)$.
Therefore,
$L^* = 0$, and $(\forall (c,d) \in C \times D)[R(c,d) = L(c,d) \wedge F(c,d) = 0]$; and
$EC_1(0,0) = 10, EC_1(0,1) = 6, EC_1(1,0) = 6, EC_1(1,1) = 2, EC_1(2,0) = 2, EC_1(2,1) = 2$;
$EC_2(0,0) = 0, EC_2(0,1) = 1, EC_2(1,0) = 0, EC_2(1,1) = 1, EC_2(2,0) = 4, EC_2(2,1) = 1.$
Here $(2,1)$ is the only Nash equilibrium; and f is efficient for the given application.[5]

[4]This rule belongs to the class of negligence rules; and is efficient for every application.

[5]This rule is also efficient for every application. The rule belongs to the class of the rule of strict liability with the defense of contributory negligence.

2 Characterization of Efficient Decomposed Liability Rules

First we formally define the condition of negligence liability.
Condition of Negligence Liability (NL): A (λ, θ) - decomposed liability rule f satisfies the condition of negligence liability iff $[[\forall p \in [0,1)][f(p,1) = (1,0)] \ \wedge \ [\forall q \in [0,1)][f(1,q) = (0,1)]]$.

In other words, a decomposed liability rule satisfies the condition of negligence liability iff its structure is such that (i) whenever the injurer is nonnegligent and the victim is negligent, the entire excess loss in case of accident is borne by the victim, and (ii) whenever the victim is nonnegligent and the injurer is negligent, the entire excess loss in case of an accident is borne by the injurer.

The most important result of this paper, stated in Theorem 1, says that, for $0 \leq \theta \leq 1$, efficient decomposed liability rules are characterized by the condition of negligence liability defined above. The following is the formal statement of the the characterization theorem:

Theorem 1 *Let $0 \leq \theta \leq 1$. A (λ, θ) - decomposed liability rule is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5) iff it satisfies the condition of negligence liability.*

The theorem is proved via three lemmas whose statements and proofs are given in the Appendix. Lemma 1 establishes that if a (λ, θ) - decomposed liability rule satisfies NL and $0 \leq \theta \leq 1$ then regardless of which permissible application, i.e., application satisfying (A1) - (A5), of the decomposed liability rule is considered, $(c^*, d^*) \in M$ constitutes a Nash equilibrium. Lemma 2 establishes that if $0 \leq \theta \leq 1$ then regardless of which permissible application of a (λ, θ) - decomposed liability rule satisfying NL is considered, all Nash equilibria will be total social cost minimizing. Lemmas 1 and 2 together thus show that if $0 \leq \theta \leq 1$ then a (λ, θ) - decomposed liability rule satisfying NL is efficient for every permissible application. Thus, given $0 \leq \theta \leq 1$, Lemmas 1 and 2 establish the sufficiency of NL for efficiency. Lemmas 1 and 2 together, in fact, establish more than the sufficiency part. To establish sufficiency part we need, for every application, only (i) all Nash equilibria to be total social cost minimizing and (ii) the existence of a Nash equilibrium. Rather than merely showing the existence of a Nash equilibrium, Lemma 1 establishes a much stronger result; namely that for every (λ, θ) - decomposed liability rule satisfying NL, $0 \leq \theta \leq 1$, the configuration of due care levels is always a Nash equilibrium.

Lemma 3 establishes that NL is a necessary condition for efficiency of any (λ, θ) - decomposed liability rule, where $0 \leq \theta \leq 1$. As a (λ, θ) - decomposed liability rule is efficient iff it is efficient for every permissible application, it follows that in order to show that NL is necessary for efficiency of any (λ, θ) - decomposed liability rule, $0 \leq \theta \leq 1$, one has to show that regardless of which (λ, θ) - decomposed liability rule violating NL one considers, $0 \leq \theta \leq 1$, one can always find a permissible application for which the decomposed liability rule in question would be inefficient.

The set of all (λ, θ) - decomposed liability rules can be partitioned into the following three subsets: (i) (λ, θ) - decomposed liability rules for which $0 \leq \theta \leq 1$ and NL is satisfied; (ii) (λ, θ) - decomposed liability rules for which $0 \leq \theta \leq 1$ and NL is violated; and (iii) (λ, θ) - decomposed liability rules for which $\theta > 1$. By Theorem 1 every (λ, θ) - decomposed liability rule belonging to subset (i) is efficient; and every (λ, θ) - decomposed liability rule belonging to subset (ii) is inefficient. This leaves only the rules belonging to subset (iii). The efficiency question for (λ, θ) - decomposed liability rules belonging to subset (iii) is completely settled by Theorem 2, given below, which says that all (λ, θ) - decomposed liability rules belonging to subset (iii) are inefficient. Thus Theorems 1 and 2 together completely settle the efficiency question for all decomposed liability rules.

Theorem 2 *Let f be a (λ, θ) - decomposed liability rule. If $\theta > 1$ then f is not efficient.*

3 The Normative Significance of the Notion of a Decomposed Liability Rule

The most important implication of the results of this paper is that if the notion of a decomposed liability rule is used to apportion liability between the victim and the injurer then the set of efficient rules becomes much larger in comparison to the set of efficient rules when the traditional notion of a liability rule is used for apportioning liability between the two parties. The enlargement of the set of efficient rules has profound implications for fulfilment of non-efficiency criteria. These implications, however, differ depending on what viewpoint one takes regarding the normative criterion of efficiency.

If one takes the viewpoint that efficiency is the only criterion that matters then of course one would be indifferent among all efficient rules; and the enlargement of the set of efficient rules would be of no consequence. While many may regard efficiency as the preeminent value, it is doubtful if anyone would seriously advance the position that there is no value other than that of efficiency which is of any relevance. If other values have some role to play, no matter how small, then enlargement of the set of efficient rules, in general, would have implications for the satisfaction of non-efficiency values. If efficiency is considered the preeminent value, i.e., dominates over all other values, then the choice of the rule for apportioning liability between the parties must be from the set of efficient rules only. From the perspective of efficiency it does not matter which rule is chosen; but from the perspective of other values different choices in general would have different implications. The important point, however, is that from the perspective of any non-efficiency value enlargement of the set of efficient rules can never make the situation worse; and might make it better. The notion of a decomposed liability rule makes the set of rules available to the society for liability apportionment much larger than what would be the case if the notion of a liability rule is used for apportioning liability. Applying the principle that if one set is a proper superset of another set then the choice from the former need never be worse than the one from the latter; it follows that, the set of rules available to the society for liability apportionment when the notion of a decomposed liability rule is used being a proper superset of the set of rules available when the the notion of a liability rule is used, the society can never be worse-off, when choosing from the set of all decomposed liability rules rather than from the set of all liability rules, regardless of the values it might decide to uphold.

If the notion of a liability rule is used for apportioning liability then, as has already been discussed, the efficiency requirement completely specifies the liability apportionment in cases where one party is negligent and the other nonnegligent. The superiority of the notion of a decomposed liability rule lies in the fact that one can satisfy the efficiency requirement as well as have some degree of freedom in specifying liability apportionment in cases where one party is negligent and the other nonnegligent. In this connection, one may, however, raise the point that if everyone behaves rationally, there is perfect information and there are no errors of any kind then it would never happen that one party behaves negligently and the other nonnegligently; and consequently the superiority of decomposed liability rules would never come into play. And therefore, the decomposed liability rules, at best, may have some relevance only in contexts where information is less than perfect, or there are measurement errors or the parties do not behave completely rationally.

The following argument, however, can be made to put forward the viewpoint that even if the outcomes which result under different rules as a consequence of interaction of rational individuals are all identical, it is not a mater of indifference as to which of these rules is selected by the society: A rule can be viewed as an embodiment of values. From this perspective choice of a particular rule is tantamount to choosing a particular set of values. Therefore, even if certain rules all yield the same outcome through interaction of rational individuals in general it would not be a matter of indifference as to which rule is selected. The only circumstance under which it would be a matter of indifference as to which rule is selected from the feasible set would be when all rules in the set are identical with respect to the relevant social values; and differ only with respect to those values which do not matter socially. An example may help illustrate the point. Under conditions of full information, no errors and rational behaviour, whether the punishment for a petty crime is mild but sufficient for deterring it altogether or is greater than the punishment for premeditated murder would not make any difference as to the actual outcome. But it is doubtful if any one would regard it as a matter of indifference as to which of the two is selected by the society.

When there are errors or other factors leading to non-attainment of efficient outcomes under liability rules; decomposed liability rules can be used to reduce the social loss due to inefficiency. For instance if the optimal loss, i.e., the loss which will take place if both parties take due care is large, court errors can lead to the injurer taking excessive care under the rule of negligence. By making the injurer liable for the optimal loss regardless of the care level taken by him or her the incentive to take excessive care on the part of the injurer can be reduced; and consequently a reduction in the amount by which the care taken by the injurer would exceed the due care might be brought about resulting in the reduction of socially wasteful costs.

Thus the significance of the notion of a decomposed liability rule is two-fold: If one views rules as embodiments of values then the scope for satisfying non-efficiency values which might be considered socially desirable, in addition to satisfying efficiency, is much broader under the decomposed liability rules than it is under the traditional liability rules. This is true regardless of whether the restrictive assumptions of complete rationality, full information and no errors hold or do not hold. If there are errors then the actual outcomes would tend to deviate from the efficient outcomes. The loss due to inefficiency however, in general, would be smaller under the decomposed liability rules than what it would be under the liability rules, as the incentives to take excessive care under the former are in general not as strong as they are under the latter when there are expectations of errors.

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Appendix

Lemma 1 *Let $0 \leq \theta \leq 1$. If a (λ, θ) - decomposed liability rule satisfies condition NL then for any arbitrary choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5), (c^*, d^*) is a Nash equilibrium.*

Proof: Let f be a (λ, θ) - decomposed liability rule satisfying NL, where $0 \leq \theta \leq 1$. Take any C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5).
Suppose (c^*, d^*) is not a Nash equilibrium. This implies:
$(\exists c' \in C)[c' + x[p(c'), q(d^*)]R(c', d^*) + \lambda F(c', d^*) < c^* + x[p(c^*), q(d^*)]R(c^*, d^*) + \lambda F(c^*, d^*)] \vee (\exists d' \in D)[d' + y[p(c^*), q(d')]R(c^*, d') + (1-\lambda)F(c^*, d') < d^* + y[p(c^*), q(d^*)]R(c^*, d^*) + (1-\lambda)F(c^*, d^*)]$.

(1.1)

First suppose $(\exists c' \in C)[c' + x[p(c'), q(d^*)]R(c', d^*) + \lambda F(c', d^*) < c^* + x[p(c^*), q(d^*)]R(c^*, d^*) + \lambda F(c^*, d^*)]$ holds. (1.2)

We first consider the case $c' < c^*$.
$c' < c^* \rightarrow x[p(c'), q(d^*)] = 1$, by condition NL. (1.3)
As $(c^*, d^*) \in M$ it follows that:
$c' < c^* \rightarrow L(c', d^*) > L^*$
$\rightarrow L(c', d^*) - \theta L^* > 0$, as $0 \leq \theta \leq 1$
$\rightarrow R(c', d^*) = L(c', d^*) - \theta L^* \wedge F(c', d^*) = \theta L^*$. (1.4)
In view of (1.3) and (1.4), (1.2) implies:
$c' + L(c', d^*) - \theta L^* + \lambda\theta L^* < c^* + x^1(1-\theta)L^* + \lambda\theta L^*$, where x^1 denotes $x(1,1)$. (1.5)
(1.5) $\rightarrow c' + d^* + L(c', d^*) < c^* + d^* + \theta L^* + x^1(1-\theta)L^*$
$\rightarrow c' + d^* + L(c', d^*) < c^* + d^* + L^*$, as $0 \leq x^1 \leq 1$.
This is a contradiction as $(c^*, d^*) \in M$, and therefore $TSC(c', d^*)$ cannot be less than $TSC(c^*, d^*)$.
This contradiction establishes that $c' < c^* \rightarrow$ (1.2) cannot hold. (1.6)

Next consider the case when $c' > c^* \wedge L(c', d^*) > \theta L^*$.
If $c' > c^* \wedge L(c', d^*) > \theta L^*$ then (1.2) implies:
$c' + x^1[L(c', d^*) - \theta L^*] + \lambda\theta L^* < c^* + x^1(1-\theta)L^* + \lambda\theta L^*$
$\rightarrow c' + x^1 L(c', d^*) < c^* + x^1 L^*$
$\rightarrow (1-x^1)c' + x^1[c' + d^* + L(c', d^*)] < (1-x^1)c^* + x^1[c^* + d^* + L^*]$
$\rightarrow (1-x^1)c' < (1-x^1)c^*$, as $x^1 \geq 0$ and $TSC(c', d^*) \geq TSC(c^*, d^*)$. (1.7)

If $(1-x^1)>0$ then $[(1.7) \rightarrow c' < c^*]$. (1.8)

If $(1-x^1)=0$ then $[(1.7) \rightarrow 0<0]$. (1.9)

$c' < c^*$ contradicts the hypothesis that $c' > c^*$ and $0<0$ is a contradiction. Therefore we conclude that:

$c' > c^* \wedge L(c', d^*) > \theta L^* \rightarrow$ (1.2) cannot hold. (1.10)

Finally consider the case when $c' > c^* \wedge L(c', d^*) \leq \theta L^*$.

If $c' > c^* \wedge L(c', d^*) \leq \theta L^*$ then (1.2) implies:

$c' + \lambda L(c', d^*) < c^* + x^1(1-\theta)L^* + \lambda\theta L^*$. (1.11)

$(1.11) \rightarrow (1-\lambda)c' + \lambda[c' + d^* + L(c', d^*)] < (1-\lambda)c^* + \lambda[c^* + d^* + L^*]$

$- \lambda L^* + x^1(1-\theta)L^* + \lambda\theta L^*$

$\rightarrow (1-\lambda)c' < (1-\lambda)c^* - \lambda L^* + x^1(1-\theta)L^* + \lambda\theta L^*$,

as $\text{TSC}(c', d^*) \geq \text{TSC}(c^*, d^*)$ and $\lambda \geq 0$

$\rightarrow (1-\lambda)c' < (1-\lambda)c^* - \lambda L^* + (1-\theta)L^* + \lambda\theta L^*$, as $0 \leq x^1 \leq 1$

$\rightarrow (1-\lambda)c' < (1-\lambda)c^* + (1-\theta)(1-\lambda)L^*$ (1.12)

If $(1-\lambda)=0$ then $[(1.12) \rightarrow 0<0$, a contradiction$]$. (1.13)

If $(1-\lambda)>0$ then $[(1.12) \rightarrow c' < c^* + (1-\theta)L^*]$. (1.14)

But we have: $L^* - L(c', d^*) \leq c' - c^*$, as $(c^*, d^*) \in M$;

and

$L(c', d^*) \leq \theta L^*$, by hypothesis.

Consequently, $L^* \leq c' - c^* + \theta L^*$

$\rightarrow c^* + (1-\theta)L^* \leq c'$, (1.15)

contradicting $c' < c^* + (1-\theta)L^*$.

In view of (1.13) - (1.15), it follows that:

$c' > c^* \wedge L(c', d^*) \leq \theta L^* \rightarrow$ (1.2) cannot hold. (1.16)

(1.6), (1.10) and (1.16) establish that (1.2) cannot hold. (1.17)

By an analogous proof one can show that:

$(\exists d' \in D)[d' + y[p(c^*), q(d')]R(c^*, d') + (1-\lambda)F(c^*, d') < d^* + y[p(c^*), q(d^*)]R(c^*, d^*) + (1-\lambda)F(c^*, d^*)]$ cannot hold. (1.18)

(1.17) and (1.18) establish the proposition.

Lemma 2 *Let $0 \leq \theta \leq 1$. If a (λ, θ) - decomposed liability rule satisfies condition NL then for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5):* $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ *is a Nash equilibrium* $\rightarrow (\overline{c}, \overline{d}) \in M]$.

Proof: Let $0 \leq \theta \leq 1$. Let f be a (λ, θ) - decomposed liability rule satisfying NL. Take any C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5).
Let $(\overline{c}, \overline{d})$ be a Nash equilibrium. $(\overline{c}, \overline{d})$ being a Nash equilibrium implies:

$$(\forall c \in C)[\overline{c} + x[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + \lambda F(\overline{c}, \overline{d}) \leq c + x[p(c), q(\overline{d})]R(c, \overline{d}) + \lambda F(c, \overline{d})] \quad (2.1)$$

and

$$(\forall d \in D)[\overline{d} + y[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + (1-\lambda)F(\overline{c}, \overline{d}) \leq d + y[p(\overline{c}), q(d)]R(\overline{c}, d) + (1-\lambda)F(\overline{c}, d)] \quad (2.2)$$

$$(2.1) \rightarrow [\overline{c} + x[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + \lambda F(\overline{c}, \overline{d}) \leq c^* + x[p(c^*), q(\overline{d})]R(c^*, \overline{d}) + \lambda F(c^*, \overline{d})] \quad (2.3)$$

$$(2.2) \rightarrow [\overline{d} + y[p(\overline{c}), q(\overline{d})]R(\overline{c}, \overline{d}) + (1-\lambda)F(\overline{c}, \overline{d}) \leq d^* + y[p(\overline{c}), q(d^*)]R(\overline{c}, d^*) + (1-\lambda)F(\overline{c}, d^*)] \quad (2.4)$$

Adding inequalities (2.3) and (2.4) we obtain:

$$\overline{c} + \overline{d} + R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) \leq c^* + d^* + x[p(c^*), q(\overline{d})]R(c^*, \overline{d}) + y[p(\overline{c}), q(d^*)]R(\overline{c}, d^*) + \lambda F(c^*, \overline{d}) + (1-\lambda)F(\overline{c}, d^*). \quad (2.5)$$

First we consider the case when $\overline{c} < c^* \ \wedge \ \overline{d} < d^*$.
$\overline{c} < c^* \ \wedge \ \overline{d} < d^* \ \rightarrow \ R(\overline{c}, \overline{d}) = L(\overline{c}, \overline{d}) - \theta L^* \ \wedge \ F(\overline{c}, \overline{d}) = F(c^*, \overline{d}) = F(\overline{c}, d^*) = \theta L^*$.
Therefore (2.5) reduces to:
$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) - \theta L^* + \theta L^* \leq c^* + d^* + \lambda\theta L^* + (1-\lambda)\theta L^*$,
as $x[p(c^*), q(\overline{d})] = 0 \ \wedge \ y[p(\overline{c}), q(d^*)] = 0$ by NL
$\rightarrow \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + \theta L^*$
$\rightarrow \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L^*$.
As TSC is minimized at (c^*, d^*), it follows that it must be the case that:
$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) = c^* + d^* + L^*$
This establishes that:
$(\overline{c}, \overline{d})$ is a Nash equilibrium and $\overline{c} < c^* \ \wedge \ \overline{d} < d^* \ \rightarrow \ (\overline{c}, \overline{d}) \in M$. (2.6)

Next we consider the case when $\overline{c} < c^* \ \wedge \ \overline{d} \geq d^*$.
If $\overline{c} < c^* \ \wedge \ \overline{d} \geq d^*$, in view of NL, (2.5) reduces to:

$$\overline{c} + \overline{d} + R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) \leq c^* + d^* + x^1 R(c^*, \overline{d}) + \lambda F(c^*, \overline{d}) + (1-\lambda)F(\overline{c}, d^*). \quad (2.7)$$

Now, by the definitions of functions R and F we have:
$(\forall (c, d) \in C \times D)[R(c, d) + F(c, d) = L(c, d)]$.
Therefore (2.7) implies:
$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x^1 R(c^*, \overline{d}) + \lambda F(c^*, \overline{d}) + (1-\lambda)\theta L^*$,
as $\overline{c} < c^* \ \rightarrow \ F(\overline{c}, d^*) = \theta L^*$
$\rightarrow \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x^1 R(c^*, \overline{d}) + \lambda\theta L^* + (1-\lambda)\theta L^*$,

as $F(c^*, \overline{d}) \leq \theta L^*$ by the definition of function F

$$\rightarrow \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + R(c^*, \overline{d}) + \theta L^* \tag{2.8}$$

Now, $R(c^*, \overline{d}) + \theta L^* = L(c^*, \overline{d}), \quad \text{if } L(c^*, \overline{d}) > \theta L^*$

$\qquad\qquad\qquad\qquad = \theta L^*, \qquad\quad \text{if } L(c^*, \overline{d}) \leq \theta L^*.$

As $\overline{d} \geq d^*$, we have $L(c^*, \overline{d}) \leq L^*$.

Also, $\theta L^* \leq L^*$, as $0 \leq \theta \leq 1$.

Therefore $R(c^*, \overline{d}) + \theta L^* \leq L^*$.

Consequently, (2.8) $\rightarrow \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L^*$.

This establishes that:

$$(\overline{c}, \overline{d}) \text{ is a Nash equilibrium} \wedge \overline{c} < c^* \ \wedge \ \overline{d} \geq d^* \ \rightarrow \ (\overline{c}, \overline{d}) \in M. \tag{2.9}$$

By an analogous argument it follows that:

$$(\overline{c}, \overline{d}) \text{ is a Nash equilibrium} \wedge \overline{c} \geq c^* \ \wedge \ \overline{d} < d^* \ \rightarrow \ (\overline{c}, \overline{d}) \in M. \tag{2.10}$$

Finally consider the case when $\overline{c} \geq c^* \ \wedge \ \overline{d} \geq d^*$.

If $\overline{c} \geq c^* \ \wedge \ \overline{d} \geq d^*$, (2.5) reduces to:

$\overline{c} + \overline{d} + R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) \leq c^* + d^* + x^1 R(c^*, \overline{d}) + y^1 R(\overline{c}, d^*) + \lambda F(c^*, \overline{d}) + (1 - \lambda) F(\overline{c}, d^*)$,

where $y^1 = y(1, 1)$.

$$\rightarrow \ \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x^1 R(c^*, \overline{d}) + y^1 R(\overline{c}, d^*) + \theta L^*, \tag{2.11}$$

as $R(\overline{c}, \overline{d}) + F(\overline{c}, \overline{d}) = L(\overline{c}, \overline{d}), F(c^*, \overline{d}) \leq \theta L^*$ and $F(\overline{c}, d^*) \leq \theta L^*$.

Now, as $R(c^*, \overline{d}) = L(c^*, \overline{d}) - \theta L^*, \quad \text{if } L(c^*, \overline{d}) - \theta L^* > 0$

$\qquad\qquad\qquad\qquad = 0, \qquad\qquad\qquad \text{if } L(c^*, \overline{d}) - \theta L^* \leq 0,$

and

$L(c^*, \overline{d}) \leq L^*$,

it follows that:

$$R(c^*, \overline{d}) \leq (1 - \theta) L^*. \tag{2.12}$$

$$\text{Similarly, } R(\overline{c}, d^*) \leq (1 - \theta) L^*. \tag{2.13}$$

In view of (2.12) and (2.13), it follows from (2.11) that:

$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L^*$,

which establishes that:

$$(\overline{c}, \overline{d}) \text{ is a Nash equilibrium and } \overline{c} \geq c^* \ \wedge \ \overline{d} \geq d^* \ \rightarrow \ (\overline{c}, \overline{d}) \in M. \tag{2.14}$$

(2.6), (2.9,), (2.10) and (2.14) establish the proposition.

Lemma 3 *Let $0 \leq \theta \leq 1$. If a (λ, θ) - decomposed liability rule is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5), then it satisfies condition NL.*

Proof: Let $0 \leq \theta \leq 1$. Suppose (λ, θ) - decomposed liability rule f violates NL. Then:
$[\exists p \in [0,1)][f(p,1) \neq (1,0)] \ \vee \ [\exists q \in [0,1)][f(1,q) \neq (0,1)]$.
Suppose $[\exists q \in [0,1)][f(1,q) \neq (0,1)]$ holds.
Suppose for $q \in [0,1)$ we have:
$f(1,q) = (x_q, y_q), y_q \neq 1$.
Let t be a positive number. As $y_q \in [0,1)$, we have $y_q t < t$. Choose a positive number r such that $y_q t < r < t$.
As $q \neq 1, (1-q) \neq 0$. Let $d_0 = \frac{r}{1-q}$
Let $0 < \epsilon, 0 < c_0$ and $0 < \delta < r - y_q t$.
Now let C, D and L be specified as follows:
$C = \{0, c_0, c_0 + \delta\}, D = \{0, qd_0, d_0, d_0 + \delta\}$,
$L(0,0) = t + qd_0 + c_0 + \epsilon + \delta, L(0, qd_0) = t + c_0 + \epsilon + \delta, L(0, d_0) = c_0 + \epsilon + \delta, L(0, d_0 + \delta) = c_0 + \epsilon + \frac{1}{2}\delta$,
$L(c_0, 0) = t + qd_0 + \delta, L(c_0, qd_0) = t + \delta, L(c_0, d_0) = \delta, L(c_0, d_0 + \delta) = \frac{1}{2}\delta$,
$L(c_0 + \delta, 0) = t + qd_0 + \frac{1}{2}\delta, L(c_0 + \delta, qd_0) = t + \frac{1}{2}\delta, L(c_0 + \delta, d_0) = \frac{1}{2}\delta, L(c_0 + \delta, d_0 + \delta) = 0$.
As $\epsilon > 0$ and $t > r = (1-q)d_0$, it follows that (c_0, d_0) is the unique total social cost minimizing configuration.
Let $(c_0, d_0) = (c^*, d^*)$.
It should be noted that the above specification of C, D, L and $(c^*, d^*) \in M$ is consistent with (A1)- (A5).
Furthermore, the specification of L is done in such a way that no inconsistency would arise even if $q = 0$.
Now, expected costs of the injurer at $(c_0, qd_0) = EC_2(c_0, qd_0)$
$= qd_0 + y_q R(c_0, qd_0) + (1-\lambda)F(c_0, qd_0)$
$= qd_0 + y_q[L(c_0, qd_0) - \theta L^*] + (1-\lambda)\theta L^*$
$= qd_0 + y_q(t + \delta) - y_q \theta\delta + (1-\lambda)\theta\delta$

$EC_2(c_0, d_0)$
$= d_0 + y[p(c_0), q(d_0)]R(c_0, d_0) + (1-\lambda)F(c_0, d_0)$
$= d_0 + y^1(1-\theta)\delta + (1-\lambda)\theta\delta$

$EC_2(c_0, d_0) - EC_2(c_0, qd_0)$
$= d_0 + y^1(1-\theta)\delta + (1-\lambda)\theta\delta - qd_0 - y_q(t+\delta) + y_q\theta\delta - (1-\lambda)\theta\delta$
$= (r - y_q t) + y^1(1-\theta)\delta - y_q\delta(1-\theta)$
$> (r - y_q t) + y^1(1-\theta)\delta - \delta$
$= (r - y_q t - \delta) + y^1(1-\theta)\delta > 0$.
This establishes that (c_0, d_0) is not a Nash Equilibrium. Consequently f is not an efficient (λ, θ) - decomposed liability rule. The proof is completed by noting that, in case $[\exists p \in [0,1)][f(p,1) \neq (1,0)]$ holds, an analogous argument shows that it is not the case that f is an efficient (λ, θ) - decomposed liability rule.

Proof of Theorem 1: Let $0 \leq \theta \leq 1$. If a (λ, θ) - decomposed liability rule satisfies the condition of negligence liability then by Propositions 1 and 2 it is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5). Proposition 3 establishes that if a (λ, θ) - decomposed liability rule is efficient for every possible choice of C, D, Π, H and $(c^*, d^*) \in M$ satisfying (A1) - (A5), then it satisfies the condition of negligence liability.

Proof of Theorem 2: Let f be a (λ, θ) - decomposed liability rule and $\theta > 1$.
Let $0 < \epsilon < \delta$. There exists a positive integer n such that:
$\frac{\epsilon}{n\delta} < \theta - 1$.
We consider the following specification of C, D and L.
$C = D = \{0, \frac{\epsilon}{3}\}$.
$L(0,0) = n\delta + \epsilon, L(\frac{\epsilon}{3}, 0) = L(0, \frac{\epsilon}{3}) = n\delta + \frac{\epsilon}{2}, L(\frac{\epsilon}{3}, \frac{\epsilon}{3}) = n\delta$.
$(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ is the unique total social cost minimizing configuration.
Let $(c^*, d^*) = (\frac{\epsilon}{3}, \frac{\epsilon}{3})$.
Now, $n\delta + \epsilon - \theta L^* = n\delta + \epsilon - \theta n\delta = n\delta[1 + \frac{\epsilon}{n\delta} - \theta] < 0$.
Therefore, $R(0,0) = R(\frac{\epsilon}{3}, 0) = R(0, \frac{\epsilon}{3}) = R(\frac{\epsilon}{3}, \frac{\epsilon}{3}) = 0$.
And therefore, $(\forall (c,d) \in C \times D)[F(c,d) = L(c,d)]$.
We have: $\lambda \leq \frac{1}{2} \ \vee \ 1 - \lambda \leq \frac{1}{2}$.
First consider the case when $\lambda \leq \frac{1}{2}$.
The expected costs of the victim at $(\frac{\epsilon}{3}, \frac{\epsilon}{3}) = EC_1(\frac{\epsilon}{3}, \frac{\epsilon}{3}) = \frac{\epsilon}{3} + \lambda n\delta$.
$EC_1(0, \frac{\epsilon}{3}) = 0 + \lambda(n\delta + \frac{\epsilon}{2})$.
$EC_1(\frac{\epsilon}{3}, \frac{\epsilon}{3}) - EC_1(0, \frac{\epsilon}{3})$
$= \frac{\epsilon}{3} + \lambda n\delta - \lambda n\delta - \lambda\frac{\epsilon}{2}$
$= \epsilon(\frac{1}{3} - \frac{\lambda}{2})$
$\geq \epsilon(\frac{1}{3} - \frac{1}{4})$, as $\lambda \leq \frac{1}{2}$
> 0.
Thus if $\lambda \leq \frac{1}{2}$ then the unique total social cost minimizing configuration $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ is not a Nash equilibrium.
If $1 - \lambda \leq \frac{1}{2}$ then an analogous argument shows that $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ is not a Nash equilibrium as the expected costs of the injurer at $(\frac{\epsilon}{3}, \frac{\epsilon}{3})$ are greater than at $(\frac{\epsilon}{3}, 0)$.

This establishes that f is not efficient.