Bank Runs, Capital Adequacy, and Line of Credit

Shubhashis Gangopadhyay*

Gurbachan Singh**

December 2003

Keywords: Bank runs, Capital adequacy, Line of credit, Subordinated debt.

JEL Classification: G21; G28

Acknowledgements: This is a considerably revised version of our discussion paper 01-04 at Indian Statistical Institute (ISI), Delhi Centre. We thank the participants at the conference on Bankruptcy and Debt Recovery Procedures on January 15-16, 2001, where a very preliminary version was first presented. Helpful comments were also given by Kose John, Arijit Sen, Oren Sussman, Dimitrios Tsomocos and Clas Wihlborg. The authors are responsible for all remaining errors.

* India Development Foundation, Gurgaon, Haryana, India

(Ph: 00-91-124-5014055, E-mail: sg@idfresearch.org)

** Jawaharlal Nehru University, New Delhi, India

(Ph: 00-91-11-26704403, E-mail: gbsingh@mail.jnu.ac.in)

Bank Runs, Capital Adequacy, and Line of Credit

Abstract

In our model, costly bank runs can occur in a bank with inadequate equity capital. The paper suggests market alternatives for undercapitalised banks. The bank can opt for a line of credit through a generalised contract that, in equilibrium, resembles pure insurance. The paper also shows how subordinated debt in a general equilibrium setup can also be used to recapitalize a bank. The major point being made in this paper is that, government intervention, beyond a regulatory role, is not necessary, especially since non-market type intervention can have other distortionary effects.

1 Introduction

The widespread banking crises in the last two decades have resulted in a number of recommendations to prevent such occurrences. The logic of these recommendations are based on the notion that banks, once set up, are prone to default on their committed deposit payments because of the mismatch between the timings of the demand for withdrawals and the returns on the assets banks create. Indeed, the possibility of such a mismatch can encourage depositors to withdraw larger amounts than they would have, if such possibilities did not exist. Therefore, one aspect of bank regulation is to ensure that depositors who do not need to withdraw at present are given enough assurance that they will be paid in the future.

This assurance to depositors can be given in a number of different ways. All of them are ways and means of ensuring that the bank has enough (claims on) liquid assets to meet all demands made by depositors. In this paper we consider four ways to provide this assurance — adequate bank equity capital, deposit insurance, lender of last resort and subordinated debt. Lack of assurance can lead to bank failures and outcomes that are not welfare maximizing. These assurances have an economic role in providing optimal outcomes and, therefore, are services that need to be properly priced.

The pricing of these assurances is an important concern (Kane, 1985; Chan, Greenbaum and Thakor, 1992; Freixas and Rochet, 1997). However, as Sleet and Smith (2000) argue, much of this literature restricts itself to a partial equilibrium analysis. Indeed, they go on to argue that, at least some of the conclusions of this literature are untenable in a general equilibrium setting. There is yet another reason why a general equilibrium setting is necessary. During the Great Depression in the USA, losses on deposits were almost 5 percent whereas the percentage of national banks which failed was somewhere between 16 and 26 percent (Calomiris and Gorton, 1991). As far as failed banks imply zero value to bank shareholders, it is in the interest of equity holders to keep the probability of bank failures as low as possible. Any policy exercise must not ignore this reality if they want to minimize the cost of regulation. In this paper we explicitly model the bank owners' behavior.

Ours is a three-period model, dated $0, 1, 2$. The private economy has two types of agents — one of whom are risk averse and the other risk neutral. There is also an institution, which could be the government, or the Central Bank, or the IMF, or indeed any financial institution in the private sector with some resources. We solve for a general equilibrium solution where a bank adds to total surplus, the risk-neutral agents find it profitable to set up a bank and become its owners by virtue of buying the shares of the bank and, the risk averse agents maximize utility by becoming depositors in the bank. The equilibrium also specifies when it is in the interest of the bank owners to "buy" assurance from the institution. Since it is a general equilibrium solution, it allows for the assurance to be properly priced.

Bhattacharya, Boot and Thakor (1998) divide the banking literature into two categories — those dealing with the asset side of banks and those dealing with the liabilities. Papers dealing with the asset side, assume risk neutral investors. Banks perform a monitoring role, which becomes too costly if individuals do it themselves. However, instead of banks, we can consider any intermediary that is equally proficient in monitoring. The defining characteristic of our bank is that it sells non-traded deposits. Other financial institutions are different from the bank in that they do not sell demand deposits. Demand deposits bind the bank to a sequential service constraint when depositors come to the bank to withdraw their deposits. For banks to be welfare improving, however, they have to organize themselves in ways that may require them to sign other contracts, in addition to demand deposits. In particular, we discuss methods of assuring the depositors of "under-capitalized" banks.

The liability side of the banking literature rests largely on Bryant (1980) and Diamond and Dybvig (1983). The Diamond-Dybvig model has identical risk averse depositors with equal access to all technologies. Investors face short run liquidity shocks in an economy where asset liquidation in an interim period is inefficient. Investing in a deposit taking intermediary improves social welfare over what can be done through a decentralized market mechanism (and autarky).

The major difference in the two groups of model lies in the risk attitude of the investors. We bring the two sides of the literature together by assuming an economy that has both risk neutral and risk averse investors. One can now consider two broad classes of contracts — when payments are contingent on the state, and those where payments are independent of the state. To risk averse agents, the latter payment schemes are more desirable than the former types. For an institution that sells only shares, these instruments could be "too" risky for risk averse investors. Risk neutral investors, on the other hand, have no inherent dislike for fluctuating returns. One can then argue that, risk neutral investors own the institutions (namely banks) that issue the deposits and take over the risk faced by the risk averse. Thus, we have a bank and its owners who are residual claimants.

Diamond and Rajan (2001) show how the sequential service constraint on banks can actually improve aggregate welfare even when all players are risk neutral. Their model is characterized by a (relationship) banker who can generate a higher liquidation value than what can be obtained by other (non-relationship) lenders to a project. The banker, however, cannot credibly commit herself, through the use of standard contracts, to use the specific skills necessary to realize this higher liquidation value. The sequential service nature of deposit contracts signed by the banker with a large number of small depositors, can force a bank run if the banker tries to renegotiate the terms of the contract. A run triggered by attempts at renegotiation harms the banker and, hence, she credibly commits to the use of her specific skill, should it be required. This allows for the creation of liquidity that enables the funding of positive net present value projects and, hence, a bank is welfare improving. In a second paper (Diamond and Rajan, 2000) show that in such a model, the bank capital structure plays an important role in determining the credibility of the banker's commitments. In particular, if the bank has too much non-deposit capital, then the banker can renege on her commitment to use her specific skill.

Our model differs from Diamond and Rajan in two important ways. First, all contracts once signed are fully enforced. Second, we have depositors who are risk averse. The fragility of banks in our paper is not for imparting credibility to any contractual commitment. Deposit contracts are like insurance contracts (a la Diamond and Dybvig, 1983) with those who can wait subsidizing those who are impatient. Since information regarding who is patient and who is not is private knowledge and, there is uncertainty about the distribution of patient and impatient depositors, the insurance solution cannot be implemented through a market mechanism (as in Diamond and Dybvig, 1983). The bank, as an institution, tries to implement the welfare-improving outcome. There is one similarity in the result of our paper and those of Diamnond and Rajan. We both have a situation where it pays the bank to ensure that even though it is “fragile”, the equilibrium is one where the fragility is not realized.

As mentioned already, we will consider four methods of providing assurance. We show that, if we use the right pricing mechanisms, all the four methods are equally adept at providing Pareto optimal outcomes. However, in terms of overall resources, a deposit insurance scheme provided through a market mechanism may dominate that of a lender of last resort. The amount of resources required for providing assurance should be an important ingredient in banking regulation. The reason why many governments find it difficult to avoid banking crises is that they, and the economy, often do not have the resources necessary to re-capitalize the banks facing liquidity problems, and hence, cannot assure the nervous depositors that they will be paid in the future. Also, theoretically, for a general equilibrium model, it is important that total resources are accounted for.

Our modeling of deposit insurance and a lender of last resort uses a general contract that can be interpreted as one or the other (or a combination of both) depending on the equilibrium nature of the contract. This differentiates our model from that of Sleet and Smith (2000) who start with universal (100%) deposit insurance and then analyze its pricing. For us deposit insurance is endogenous and the bank decides how much insurance coverage to buy. The extent of the facilities to be drawn upon from the lender of last resort is also endogenous. In Sleet and Smith, however, insolvent banks utilize the discount window of the Central Bank. As Stevens (2000) points out, this is a serious departure from the actual operation of the discount window system, where only solvent banks are allowed to use this facility. In our model, therefore, solvent banks use the 'discount' window.

Gorton and Winton (2000) emphasize that it is difficult for banks to raise capital to meet capital adequacy requirements. Moreover, the regulators may find that, given the difficulties faced by banks (particularly in times of recession) and the ramifications of meeting capital adequacy requirements (for example, a squeeze on credit), it is optimal to follow a policy of "forbearance". The regulators may let the banks function with inadequate capital because it may be too costly to conform to the capital adequacy norms. However, we feel that one needs a general equilibrium framework to see if it is desirable to increase or decrease equity capital. We provide such a general equilibrium framework.

There is a widespread belief that banks are inherently unstable and that banks need to be regulated. In Diamond and Dybvig (1983), following Bryant (1980), such panic run is an equilibrium in models with multiple equilibria. More recently, Allen and Gale (1998) showed that, under some conditions, bank runs result in an optimal allocation, and under other conditions where bank runs are not optimal, government intervention helps.

On the other hand, Jacklin (1987), McCulloch and Yu (1998), and Gangopadhyay and Singh (2000) show that optimal inter-temporal allocation can be obtained without making banks run-prone and hence, government intervention is not required. Jacklin (1987) uses equity-like instrument, McCulloch and Yu (1998) uses contingent bonus contract and Gangopadhyay and Singh (2000) uses both equity and deposits, and shows that if there is adequate equity with a bank, then depositors have an assurance and hence, panic runs can be avoided. If equity capital with the commercial banks is inadequate, then also runs can be avoided but full insurance cannot be provided (a la Diamond and Dybvig, 1983) to the depositors.

The plan of the paper is as follows. Section 2 describes the model and considers the benchmark case of *adequate* equity capital. Section 3 considers the case of a bank that is undercapitalized, given its deposits. We consider two sets of alternatives in such a situation. First, we consider a line of credit that is set up in such a way that, depending on what the equilibrium looks like, it can be interpreted either as deposit insurance or, as a facility provided by the lender of last resort. Finally, in section 4, we consider subordinated debt as an alternative to equity capital. Section 5 concludes.

2 The Basic Model and the Benchmark Case

This is a three period model, $0, 1, 2$. There is a continuum of agents in the interval $[0, 1]$. Agents are either risk averse, or risk neutral. θ proportion of agents are risk averse, and

$1-\theta$ are risk neutral. Each agent, whether risk averse or risk neutral, can be either of type 1, or of type 2. Type 1 agents derive utility from consumption in period 1 only, and type 2 agents from consumption in period 2 only. In period 0, each agent faces a probability t of being type 1. An alternative interpretation is that t proportion of agents will be of type 1 and $1-t$ proportion will be of type 2. Table I describes the agent classification in period 1.

Table I about here

There is aggregate uncertainty in t. Thus, the value of t that will obtain in period 1 is not known at the time of contracting in period 0. However, everyone knows, in period 0, the distribution $F(t)$ from which t will be drawn. We make the following assumption:

A.1: $F(t)$ *is uniform, on* $[0, 1]$.

While the distribution of t is common knowledge in period 0, the type of an agent in period 1 is private information to the agent.

Each risk averse agent has an endowment of 1 unit in period 0 and nothing in other periods. Each risk neutral agent has an endowment of K units in period 0 and nothing in any other period. Thus, the total endowment in the economy in period 0 is $\theta + (1-\theta)K$. Let

$$\theta' \equiv \frac{1-\theta}{\theta} \tag{1}$$

The technology is constant returns to scale, available to everyone and, the long term return is greater than the short term one. For each unit of resource invested in period 0, the gross return is R (> 1) in period 2, i.e., in the long term. In the short term, period 1, the gross return is 1. Pure storage is also possible. It is costless, but yields a net return of zero, similar to the short term return. Also observe that, there is no uncertainty in the technology.

Let c_{ij} denote the consumption of a type i agent ($i = 1, 2$) in period j ($j = 1, 2$). Given our assumption on the consumption requirements of agents, c_{12} or c_{21} are irrelevant. The superscript a will denote the risk averse agents, while n will denote the risk neutral ones. The expected utility in period 0 is

$$EU^a = \int_0^1 [tu(c_{11}^a) + (1-t)\rho u(c_{22}^a)]dF(t) \tag{2}$$

$$EU^n = \int_0^1 [tc_{11}^n + (1-t)\rho c_{22}^n]dF(t) \tag{3}$$

where ρ is the discount factor, $0 < \rho < 1$.

A.2: $\rho R > 1$.

A.2 guarantees that agents prefer to be type 2 or, the long term returns are sufficiently high. For risk averse agents, the issue is similar to the problem of insurance. Being type 2 is a 'win' situation, while being type 1 is a 'loss'. However, since the information regarding types is private, an insurance market for risk averse agents will fail. However, a bank can substitute for an insurance mechanism (Diamond and Dybvig, 1983). The amount of risk neutral capital, measured by $(1-\theta)K$, will play a crucial role in determining the extent of such insurance that can be provided by the bank (Gangopadhyay and Singh, 2000).

A bank is an institution that can sell shares and (demand) deposits. These are issued in period 0. Deposit claims in any period are senior to claims by the shareholders in that period. For each unit invested in deposits, an agent receives either r_1 in period 1 and zero in period 2, or zero in period 1 and r_2 in period 2. Shares are long term assets (irredeemable in period 1), while deposits can be liquidated in period 1 if the depositor so wishes. The banks can offer dividend v_1 per unit of equity capital in period 1. In period 2, the residual with the bank is distributed to shareholders.[1] For getting explicit solutions we will use another assumption in this paper.

A.3: $u^a(c_{ij}) = (1-s)(c_{ij})^{1-s}$, $s > 1$, $i = 1, 2$, *and* $j = 1, 2$.

In this setup, Gangopadhyay and Singh (2000) calculate an amount $K_0 > 0$ such that if $K = K_0$, the Diamond-Dybvig first best insurance outcome can be implemented through a bank that has zero probability of a run. Any $K < K_0$ will result in the bank having a positive probability of a run, were it to attempt at implementing the first best insurance outcome. The amount $(1-\theta)(K_0 - K)$ can be used as a measure of the degree of bank undercapitalization when the bank is trying to support the first best return to depositors.[2]

Suppose all risk-averse agents are depositors and risk-neutral agents are equity holders in the bank.[3] Then, in period 0, the bank gets θ from the risk-averse depositors and $(1-\theta)K$ from those who buy equity in the bank. In the beginning of period 1, given the technology, the bank has a total of $\theta + (1-\theta)K$. In period 1, the bank has to pay an amount r_1 to each depositor who chooses to withdraw in period 1. If only the type 1 depositors are withdrawing deposits, the amount paid to depositors in period 1 is $r_1\theta t$. If the bank had also committed

[1] For completeness, we need to state one more assumption which is that bank shares are non-transferable in period 1. This is to prevent disintermediation (for details, see Jacklin, 1987; Haubrich and King, 1990; McCulloch and Yu, 1999, and Gangopadhyay and Singh, 2000). There is an analogous assumption in Diamond and Dybvig (1983) as well. By assumption, agents deposit their entire endowment in the bank.

[2] In Diamond and Dybvig (1983), K was zero and so the bank was always undercapitalized and a run was always possible (Gangopadhyay and Singh, 2000).

[3] We will show later that in equilibrium this is what will happen.

itself to a dividend payout of v_1, then it will have to pay $v_1 K$ to each equity owner. Note that dividend payouts are independent of whether equity owners are of type 1 or type 2.. Thus, if type 2 depositors are willing to wait, then the bank's total payout in period 1 is $r_1\theta t + v_1(1-\theta)K$.

The resources with the bank in period 2 are the net amount left with the bank after period 1 payouts, times the return R. For type 2 depositors to keep their money in the bank in period 1, they must be confident that the bank will have enough resources in period 2 to give them their r_2. This implies that the residual with the bank in period 2, $Z(t)$, must satisfy the following condition:

$$Z(t) = [\theta + (1-\theta)K - r_1\theta t - v_1(1-\theta)K]R - r_2\theta(1-t) \geq 0 \tag{4}$$

Z is a function of t. Since agents do not know the realization of aggregate t, for this bank to be run-proof, it must be that $Z(t) \geq 0$ for all possible t. If $Rr_1 - r_2 > 0$, then $Z(t) \geq 0$ implies

$$t \leq \frac{(1+\theta' K)R - v_1\theta' KR - r_2}{r_1 R - r_2} \equiv \underline{t}. \tag{5}$$

If $\underline{t} \geq 1$, then $Z(t) \geq 0$ for all possible values of t.

For a non-trivial solution, all players associated with the bank must do so voluntarily — i.e., each must get from the bank a utility, or payoff, that is no worse than what may be offered by some other financial institution. In the absence of any kind of financial intermediation, a risk averse agent can invest in the technology and get

$$t^e u(1) + (1-t^e)\rho u(R) \equiv \underline{A},$$

where $t^e \equiv E(t) \equiv \int_0^1 t dF(t)$. Alternatively, if a risk averse agent has access to any non-bank institution, then her reservation utility is given by A. Observe that for the non-bank institution to be feasible, $A \geq \underline{A}$.

In addition to the risk averse depositors, the return on equity investment to risk neutral agents in the bank must also be no less than their alternative earning opportunity. If each risk neutral agent invests on her own in a project (autarky), then she gets

$$[t^e.1 + (1 - t^e)\rho R]K \equiv \underline{B}.$$

Gangopadhyay and Singh (2000) showed that risk neutral agents can always contract with themselves (without combining with the risk averse agents) and obtain a return

$$B = \rho RK > \underline{B}.$$

For the risk neutral capital owners to set up a bank and provide equity capital, they must get at least this level.

The no run bank's problem can be stated as[4]

P1: $\max EU^n$ subject to (C1) $EU^a \geq A$ and (C2) $\underline{t} \geq 1$.

Let us consider EU^n. For this, we need to consider $c^n_{11}(t)$ and $c^n_{22}(t)$. The bank pays out dividend to all equity holders in period 1. Type 1 equity holders sell their ex-dividend shares and buy goods. Type 2 equity holders sell their goods and buy ex-dividend shares in a competitive market. After solving for equilibrium price of ex-dividend shares, it can

[4]In Gangopadhyay and Singh (2000) the problem was to maximize the utility of depositors given that the risk neutral equity owners get a minimum expected return on their equity investment in the bank. This section is considering a dual version of the problem stated in Gangopadhyay and Singh (2000).

be shown that[5]

$$c^n_{11}(t) = \frac{v_1 K}{t} \tag{6}$$

and

$$c^n_{22}(t) = \frac{Z(t)}{(1-t)(1-\theta)} \tag{7}$$

The intuition is simple. In period 1, the dividend pay-out is $v_1 K$ and there are t type 1 equity holders. So each type 1 equity holder consumes $\frac{v_1 K}{t}$. In period 2, the residual pay-out is $Z(t)$ and there are $(1-t)(1-\theta)$ type 2 equity holders. Hence, each type 2 equity holder gets $\frac{Z(t)}{(1-t)(1-\theta)}$ in period 2.

Using A.1, (4), (6) and (7) in (3), we get

$$EU^n = \rho RK + \frac{\rho}{\theta'}\Big\{(1 - r_1 t^e)R - r_2(1-t^e)\Big\} - v_1(\rho R - 1)K \tag{8}$$

Diamond and Dybvig (1983) had considered maximization of EU^a subject to zero profit condition of insurer. Recall that, in our set-up, the reservation utility of the risk neutral agents (the insurers) is ρRK. Hence, it follows from (8) that in equilibrium, the term in braces is equal to zero since $v_1^* = 0$ as a result of A.2 i.e. $\rho R > 1$. Hence, in equilibrium,

$$r_2 = \frac{(1 - r_1 t^e)R}{1 - t^e}$$

Maximization of EU^a as given in (2) subject to this zero profit condition yields

$$u'(r_1) = \rho R u'(r_2)$$

The zero profit condition and the necessary condition for utility maximization give us two equations in the two unknowns r_1 and r_2. Using A.1 and A.3, we can solve to get

$$r_1 = \frac{2R}{Q+R}$$

[5]See Jacklin (1987), or Gangopadhyay and Singh (2000).

$$r_2 = Qr_1$$

where $Q \equiv (\rho R)^{(1/s)}$. Substituting these values of r_1 and r_2 in (2), we get the maximized utility value. Let us denote this by $\overline{A}$. Note that $\overline{A} > \underline{A}$.

In this paper, we consider the dual of the above problem. In other words, we consider maximization of EU^n subject to $EU^a \geq A$. We can now state our first result, which sets the benchmark for later analysis.

Proposition 1: *Let A.1 - A.3 hold. Define*

$$K_0(A) \equiv \frac{1}{\theta'}\left\{\left[\frac{2A}{(1-s)(1+\rho Q^{1-s})}\right]^{\frac{1}{1-s}} - 1\right\}, \tag{9}$$

$$\overline{A} \equiv \frac{(1-s)(1+\rho Q^{1-s})}{2}\left[\frac{R+Q}{2R}\right]^{(s-1)}. \tag{10}$$

Assume that $\underline{A} \leq A \leq \overline{A}$. *If* $K \geq K_0(A)$, *then a Nash Equilibrium exists in which risk averse agents buy deposits and risk neutral agents buy equity. This equilibrium has the following properties:*

$$v_1^*(A) = 0$$

$$r_1^*(A) = \left[\frac{2A}{(1-s)(1+\rho Q^{1-s})}\right]^{\frac{1}{1-s}} = 1 + \theta' K_0(A) \tag{11}$$

$$r_2^*(A) = Qr_1^*(A)$$

The bank is run-proof.

Proof: Please see the Appendix.

From the proof, we know that

$$u'(r_1) = \rho R u'(r_2) \tag{12}$$

It is easy to check that $1 < r_1^*(A) < r_2^*(A) < R$, given that $\underline{A} \leq A \leq \overline{A}$. The intuition for an upper bound on A is that the expected utility of the equity holder cannot be less than $B = \rho RK$. The full information maximum utility level for risk averse depositors is given by $\overline{A}$ which was the focus of analysis in Diamond and Dybvig (1983). It is also easy to check that $\frac{\partial r_1^*(A)}{\partial A}$, $\frac{\partial^2 r_1^*(A)}{\partial A^2}$ and $\frac{\partial K_0(A)}{\partial A}$ are all positive. Thus the capital requirement of a bank increases as A increases. Finally, note that $r_1(\overline{A}) = \frac{2R}{Q+R}$.

Before we complete this section, let us look more closely at the characteristics of this no-run solution. Observe that for a zero probability of a run, (C2) must hold, i.e., $\underline{t} \geq 1$. From (5), this means

$$\begin{aligned} (1 + \theta' K)R - v_1\theta' KR - r_2 &\geq r_1 R - r_2 \\ \Leftrightarrow 1 + \theta' K - v_1 \theta' K &\geq r_1 \end{aligned}$$

With $v_1^* = 0$, this means that for a no-run bank, r_1 cannot be more than $1 + \theta' K$. Thus, we have two conditions on r_1; this no-run condition and the inter-temporal rates of return condition given by (12). While (12) is a necessary condition, the absolute value of r_1 is determined by the value of A, the minimum utility that the depositors must be guaranteed by the bank. $K_0(A)$ is the minimum value of K that allows both (5) and (12) to be satisfied (see (11)).

The question, therefore, is: what happens if $K < K_0(A)$? Gangopadhyay and Singh (2000) had shown that even in such a case, a no-run bank is always optimal. However, equation (12) is no longer satisfied, i.e., depositors have a no-run bank but cannot attain a full insurance outcome (a la Diamond and Dybvig, 1983). This non-fulfillment of the inter-temporal condition for risk averse depositors implies a 'distortion'. If the bank had one more unit of capital, it could raise r_1 and reduce r_2 in such a way that the depositors are no worse off but the residual equity owners get a higher expected return. This motivates the equity owners to raise additional capital to solve this problem of 'undercapitalization' of the bank. We formalize this in the next section.

3 Line of Credit

Assume that $K < K_0(A)$. The bank enters into a contract in period 0 with a supplier of line of credit. The contract is as follows. In period 1, the bank can borrow, if it wants, any amount up to a pre-specifed limit, say $(1-\theta)x$. In period 1, the bank, therefore, has access to not only the proceeds of its own liquidated investment, but also what it borrows from the supplier of the line of credit.

To buy the credit facility, the bank pays a price p per unit amount of credit coverage. If the bank buys the right to borrow a maximum amount of $(1-\theta)x$, it pays a price $p(1-\theta)x$. This facility helps the bank in reducing, if not avoiding, the extent of pre-mature liquidation of its long term asset. For each unit borrowed by the bank in period 1, the bank has to pay back in period 2 a gross amount (inclusive of interest) r_C to the provider of the credit. Thus, the contract can be denoted (x, p, r_C).

The bank has to make contracted payments to three groups of agents in period 2, the creditor, the type 2 depositors and the equity holders. We assume that the creditor is the senior most, followed by the type 2 depositors and, finally, the equity owners get the residual, if any[6].

[6] In the next section, we consider the case where the creditor is junior to depositors

Usually, a supplier of line of credit is the Central Bank. As Stevens (2000) points out[7], in most cases where the Central Bank performs this function, the banks are usually solvent. The seniority becomes an issue only when the bank does not have adequate resources to pay back all its contractual claims. One major difference with the role of the Central Bank and our modeling of the supplier of line of credit is that, we are looking for market solutions, while the Central Bank's role is not necessarily commercial in nature.

Observe that, $p \geq 0$. If $p < 0$, for each unit of credit coverage, the bank *gets* an amount p in period 0. The bank can always choose a large enough credit coverage to make up the shortage of $K_0(A) - K$. If it does, it will have, in period 0, $K(1-\theta) - p(1-\theta)x \geq K_0(A)$, will ensure a run-proof bank, will never have to use the credit line in period 1, and therefore, the creditor will always lose the amount $p(1-\theta)x$. So, $p < 0$ can never be an equilibrium for the provider of credit.

We need to specify what are the alternatives to the provider of the line of credit. We will assume that it is risk neutral and has no need for liquidity in period 1.

A.4: *The provider of the line of credit has a reservation utility equal to* $\bar{U}^C = \rho RW$, *where* W *is its period* 0 *endowment.*

If EU^C is the payoff to the creditor, from the contract (x, p, r_C), then we must have

$$EU^C \geq \rho RW = \overline{U}^C \tag{13}$$

[7]'Actual Federal Reserve discount window lending is made to solvent banks and always collateralized' (p. 576)

Define t_C to be such that $\theta r_1 t_C = (1-\theta)x$. Thus,

$$t_C \equiv \frac{\theta' x}{r_1} \tag{14}$$

Observe that, as long as $t \leq t_C$, the bank need not liquidate any of its own assets. Instead, it borrows $r_1\theta t$ to pay the type 1 depositors (if $t \leq t_C$) and repays this amount with interest, i.e. it pays $r_C(\theta r_1 t)$ to the creditor in period 2.

What happens if $t > t_C$? In this case, type 1 depositors have to be paid $\theta r_1 t > (1-\theta)x$ in period 1. The bank borrows $(1-\theta)x = \theta r_1 t_C$, and liquidates its project to the extent of meeting the remaining withdrawal requirement. So it liquidates its project to get $\theta r_1(t - t_C)$.

Compared to the earlier section, here, the type 2 depositors have to worry about an additional factor. Earlier, they needed to be convinced that the bank's resources in period 2, after paying off type 1 depositors in period 1, were sufficient to cover period 2 withdrawals of deposits. Now, there is an additional claim on the bank in period 2 and this claimant is senior to type 2 depositors. So, to prevent a run by type 2 depositors, we need for all t,

$$\begin{aligned} r_2\theta(1-t) \quad &\leq \quad \{\theta + (1-\theta)K - p(1-\theta)x - \max[0, r_1\theta(t - t_C)] \\ &\qquad -v_1(1-\theta)K\}R \quad - r_C\theta r_1 \min[t, t_C] \end{aligned} \tag{15}$$

Resources with the bank in period 1 include what it had mobilized in period 0 by way of deposits, θ, and equity, $(1-\theta)K$, minus what it had paid for getting the line of credit, $p(1-\theta)x$. It will have to pay, however, only when $t > t_C$. Condition (15) ensures that the bank has enough resources in period 2 to pay the promised pay-out to type 2 depositors. Observe that, if the bank is run-prone at low t, it will be run-prone at high t. Therefore, if we rule out a run at all $t > t_C$, we will have a bank with no run. Rewrite (15) as

$$t \leq \frac{[1 + \theta' K(1 - v_1) + \theta' x(1 - p)]R - r_2 - r_C \theta' x}{r_1 R - r_2} \equiv T, \quad r_1 R - r_2 > 0 \tag{16}$$

Thus, for a no-run bank, we will need T to be greater than or equal to 1.

Credibility of the bank to the depositors cannot be maintained unless they are confident that the supplier of credit can honor its commitments. The creditor must have enough resources in period 1 to service the credit facility. In period 1, the creditor has own wealth, W and the fees from the bank, $p(1 - \theta)x$, collected in period 0. We need

$$\begin{aligned} & W + p(1 - \theta)x \geq (1 - \theta)x \\ \Rightarrow \quad & W \geq (1 - \theta)(1 - p)x \end{aligned} \tag{17}$$

Observe that the minimum W required is not only a function of the credit coverage $(1-\theta)x$, but also of the price, p, to be paid in period 0.

Let us consider the expected return to the supplier of the line of credit, when it has enough W to extend a credit line that it can credibly honor, and the bank is able to pay back the creditor all amounts borrowed in period 1.

$$\begin{aligned} EU^C \quad = \quad & \rho \Big\{ \int_0^{t_C} r_C \theta r_1 t dF(t) + \int_{t_C}^1 r_C \theta r_1 t_C dF(t) \\ & + \int_0^{t_C} (W + p(1 - \theta)x - \theta r_1 t) R dF(t) \\ & + \int_{t_C}^1 (W + p(1 - \theta)x - (1 - \theta)x) R dF(t) \Big\} \end{aligned} \tag{18}$$

The first two expressions in the braces are what it gets as repayment from the bank for various realizations of t. The last two terms are what it gets from its residual wealth, $W + p(1 - \theta)x - \min\{\theta r_1 t, \theta r_1 t_C\}$, for different t. Observe that, the way we have written

(18) implies that there is no default on the amount borrowed from the line of credit. This is consistent with our assumption that the bank's creditor is senior to the type 2 depositors in period 2. If we allowed for default, this would mean that there is not enough money in the bank (in period 2) to pay the supplier of credit and hence, nothing for type 2 depositors. This is inconsistent with a bank with no run possibility.[8] Using (13) in equation (18), we can write

$$EU^C = \overline{U}^C + \rho(1-\theta)x\left\{pR - \frac{R - r_C}{t_C}\left[t^e - \int_{t_C}^{1}(t - t_C)dF(t)\right]\right\} \tag{19}$$

Now let us consider the equity holders. The residual, $Z(t)$ with the bank in period 2 after paying off depositors and creditor is given by

$$\begin{aligned} Z(t) &= [\theta + (1-\theta)K - p(1-\theta)x - v_1(1-\theta)K]R \\ &\quad -r_2\theta(1-t) - r_C\theta r_1 t, \qquad 0 \le t \le t_C \end{aligned}$$

and,

$$\begin{aligned} Z(t) &= [\theta + (1-\theta)K - p(1-\theta)x - \theta r_1(t - t_C) - v_1(1-\theta)K]R \\ &\quad -r_2\theta(1-t) - r_C\theta r_1 t_C, \qquad t_C < t \le 1 \end{aligned}$$

$$\begin{aligned} \Rightarrow \int_0^1 Z(t)dF(t) &= \int_0^{t_C}\Big\{[\theta + (1-\theta)K - p(1-\theta)x - v_1(1-\theta)K]R \\ &\quad -r_2\theta(1-t) - r_C\theta r_1 t\Big\}dF(t) \\ &\quad + \int_{t_C}^{1}\Big\{[\theta + (1-\theta)K - p(1-\theta)x - \theta r_1(t - t_C) \\ &\quad -v_1(1-\theta)K]R - r_2\theta(1-t) - r_C\theta r_1 t_C\Big\}dF(t) \end{aligned}$$

[8]In the next section, we consider the case where (non-deposit) debt claims on the bank could be risky.

After using A.1 and (19), we get

$$\int_0^1 Z(t)dF(t) = (1-\theta)(1-v_1)RK + \theta\{(1-r_1t^e)R - r_2(1-t^e)\} - \frac{EU^c - \overline{U}^c}{\rho} \tag{20}$$

Using A.1, (6), (7) and (20) in (3), we get

$$EU^n = \rho RK + \frac{\rho}{\theta'}\{(1-r_1t^e)R - r_2(1-t^e)\} - \frac{EU^c - \overline{U}^c}{1-\theta} + v_1K(1-\rho R) \tag{21}$$

In what follows, we will denote the solution in this section by putting a hat over the variable. The endogenous variables are $r_1(A)$, $r_2(A)$, $v_1(A)$, $x(A)$, $p(A)$ and $r_C(A)$.

Proposition 2: *Let A.1 - A.4 hold. Also, let* $K < K_0(A)$, $\underline{A} \leq A \leq \overline{A}$ *and the creditor have unlimited wealth. Suppose the credit contract* (x, p, r_C) *satisfies*

$$x \geq \frac{[K_0(A) - K]R}{R(1-p) - r_C}$$
$$p = \frac{(R - r_C)}{R}\left(1 - \frac{t_C}{2}\right)$$

where $t_C = (\theta' x)/r_1$. *Then,* $\hat{v}_1(A) = v_1^*(A) = 0$ *and* $\hat{r}_j(A) = r_j^*(A)$, $j = 1, 2$, *(*$r_j^*(A)$ *as given in Proposition 1) is a Nash Equilibrium in which risk averse agents buy deposits and risk neutral agents buy equity and the bank is run-proof.*

Proof: Please see the Appendix.

Observe that in this Proposition, we have assumed that the supplier of credit has sufficient wealth. We have also given a relationship between p and r_C, without giving explicit solutions. We will now argue that the creditor's wealth requirement is closely linked to the exact values of p and r_C.

Earlier in this section we have argued that $p \geq 0$. We will now argue that $p > 0$ for the solution in Proposition 3. If $p = 0$, then from (19), we must have $r_C \geq R$ for EU^c to be at least as much as $\bar{U}^c$. This is immediate if we evaluate the term in braces in (19). This term must be non-negative for $EU^C \geq \overline{U}^C$. With $p = 0$, this implies that the term in square brackets must be non-positive if $R \geq r_C$. However, the term in square brackets is always positive as long as $t_c > 0$. It follows, therefore that, if $p = 0$, then $r_c \geq R$. But then, from (16), we have

$$\begin{aligned} T &\geq 1 \\ \Rightarrow &\quad (1 + \theta' K)R - r_2 - (r_C - R)\theta' x \geq r_1 R - r_2 \\ \Rightarrow &\quad (1 + \theta' K) \geq r_1 \end{aligned}$$

which is not possible if $r_1 = r_1^*(A) = 1 + \theta' K_0(A)$, since $K_0(A) > K$ (see Proposition 1). Since, in Proposition 2, $\hat{r}_1 = r_1^*$, we have $p > 0$. We will now argue that the creditor requires the least amount of wealth when r_C is chosen to be zero.

Corollary 1: *Consider the contract* $(\hat{x}, \hat{p}, \hat{r_C})$, *where*

$$\hat{x} = \left[\frac{2(K_0(A) - K)r_1^*}{\theta'}\right]^{\frac{1}{2}} \tag{22}$$

$$\hat{p} = 1 - \frac{\theta' \hat{x}}{2r_1^*} \tag{23}$$

$$\hat{r}_C = 0 \tag{24}$$

Then, if $W \geq (1 - \theta)(K_0(A) - K) \equiv \underline{W}$, *then the solution in Proposition 3 can be implemented. For any* $r_C > 0$, *wealth requirement of the creditor is strictly greater than* $\underline{W}$.

Proof: Please see the Appendix.

The Corollary is an interesting result. First, it argues that any credit line with positive rates of interest (r_C) requires more capital than is otherwise necessary to have run-free banking. Second, $r_C = 0$ and $p > 0$ is somewhat like a bank buying insurance. It pays a premium p per unit of coverage. The maximum coverage bought by the bank is $(1 - \theta)x$; the actual 'insurance' claim, $r_1 \theta t$, is determined by the realization of t. But, unlike in government deposit insurance, the provider of this insurance is a private agent, with a reservation utility that it can obtain in the market. It need not provide any subsidies to the bank as is often the case in government sponsored insurance schemes.

To recapitulate, we have demonstrated the existence of run-proof banks with additional capital that is fairly priced. We have shown how a (right to) borrowing facility collapses to an insurance mechanism, and this facility can be bought and sold in the market. The government can step in only if such an agent does not exist. Even when the government provides this facility, it needs to have the requisite credibility to give this assurance. In most of the literature, this credibility is associated with the government's ability to tax. We are arguing that having enough capital is sufficient to provide this facility. More importantly, if this capital is fairly priced, the government can raise it in the market.

4 Subordinated Debt

In section 2, we demonstrated why and how run-proof banks operate without any deposit insurance when investors are atomistic and there is enough risk-neutral capital. In the previous section, we considered how an undercapitalised bank can sign market contracts to prevent a bank run. Recall that we assumed that this agent (a market operative and external to the bank) held claims (on the bank) that were senior to those of the depositors. In this section, we again consider an external agent but with two differences. We assume now that this agent has claims that are junior to those of the depositors. The other difference is that we assume that this external agent lends to the bank in period 0. The fact that this agent is junior to the depositors has an important implication. The loan facility utilized by the bank in the previous section had zero default probability. The (non-deposit) bank debt in this section is risky debt.

As in the previous section, we start by assuming that atomistic risk-neutral investors do not have adequate capital. For any alternative opportunity A, of risk-averse depositors, a bank needs equity capital of the amount $K_0(A)$, but the atomistic bank equity owners have $K < K_0(A)$. To have a no-run bank, one has to look at an alternative to capital adequacy.

Consider an agent, S, who has an endowment, say V, in period 0. Agent S is risk neutral and has no need for liquidity in period 1. In other words, S is always patient and willing to wait till period 2 to consume. This implies that S can always get a utility ρRV by investing in the technology, since she will never want to liquidate in period 1. Formally, we have:

A.4′: *Agent S is always assured of, in period* 0, *an expected utility of* ρRV, *where* V *is the period* 0 *endowment of S. This is the reservation utility of S, denoted* $\bar{U}^S$.

Consider the following contract between agent S and the bank. In period 0, S lends an amount $(1-\theta)L$ ($\leq V$) to the bank for two periods. The bank repays agent S in period 2 with interest. Let D be the amount to be repaid by the bank in period 2 per unit of borrowing in period 0, subject to solvency. We assume that agent S is junior to depositors but senior to equity holders. Assuming that type 1 depositors and type 2 depositors withdraw in period

1 and in period 2 respectively, and that the bank pays a dividend to all equity holders, the residual with the bank in period 2 is given by

$$[\theta + (1-\theta)K + (1-\theta)L - r_1\theta t - v_1(1-\theta)K]R - r_2\theta(1-t)$$

The bank is supposed to pay $D(1-\theta)L$ in period 2 to agent S. This is possible only if $D(1-\theta)L$ is less than or equal to the residual with the bank. Suppose there exists a value t_0, $0 \leq t_0 \leq 1$, such that

$$D(1-\theta)L = [\theta + (1-\theta)K + (1-\theta)L - r_1\theta t_0 - v_1(1-\theta)K]R - r_2\theta(1-t_0)$$

or,

$$t_0 \equiv \frac{[1+\theta' K(1-v_1)]R - r_2 + (R-D)\theta' L}{r_1 R - r_2}, \tag{25}$$

Then, assuming that $r_1 R - r_2 > 0$, the amount actually paid to agent S in period 2 is given by

$$Y = \begin{cases} D(1-\theta)L, & \text{if } 0 \leq t \leq t_0, \\ [\theta + (1-\theta)K + (1-\theta)L - r_1\theta t - v_1(1-\theta)K]R - r_2\theta(1-t), & \text{if } t_0 < t \leq 1. \end{cases}$$

Recall that the only uncertainty in our model is that t is stochastic. So there is a cut-off value for t such that for low values of t, agent S gets the full amount due to her and for high values of t, agent S gets the residual in period 2 and equity holders get zero.

Now we have two kinds of debts — the demand deposits held by atomistic agents and the debt held by agent S. The distinction between the two is that the former can be, as the name suggests, withdrawn on demand whereas the debt held by agent S can be redeemed in period 2 only. The other distinction is that demand depositors have seniority over agent S in their claims on the bank. Henceforth, unless otherwise specified, a deposit refers to a demand deposit held by an atomistic (risk averse) agent.

We will first develop the formal description assuming that there is no bank run and then show under what conditions this is true. For the bank (with debt capital) to offer fixed rates of interest and to avoid a run on the bank in period 1 by type 2 depositors, it is necessary and sufficient that $\forall\, t$

$$\begin{aligned} r_2\theta(1-t) &\leq [\theta + (1-\theta)K + (1-\theta)L - r_1\theta t - v_1(1-\theta)K]R \\ (1-\theta)L &\leq V \end{aligned}$$

The first condition says that the promised pay-out in period 2 must be less than or equal to the resources with the bank in period 2 after paying out to type 1 depositors and the dividend to all the equity holders in period 1. The second condition is the resource constraint of agent S. It says that the amount lent by agent S in period 0 cannot exceed the endowment of agent S in period 0. We can rewrite the first condition as

$$t \leq \frac{[1+\theta'(K(1-v_1)+L)]R - r_2}{r_1 R - r_2} \equiv t_L, \quad r_1 R - r_2 > 0 \tag{26}$$

Let EU^S denote the expected utility of S.

$$\begin{aligned} EU^S &= \rho\Big\{(V-(1-\theta)L)R + \int_0^{t_0} D(1-\theta)L dF(t) \\ &\quad + \int_{t_0}^{1} \{[\theta + (1-\theta)K + (1-\theta)L - r_1\theta t - v_1(1-\theta)K]R \\ &\quad - r_2\theta(1-t)\} dF(t)\Big\} \end{aligned} \tag{27}$$

The first term within the bigger braces is the return S gets from her wealth directly invested in the long-term technology. The other terms are what she gets from her loans to the bank. Equation (27) can be reduced to

$$\begin{aligned} EU^S &= \overline{U}^S + \rho\theta\{(1-r_1 t^e)R - r_2(1-t^e)\} + \rho(1-\theta)RK(1-v_1) \\ &\quad - \rho\int_0^{t_0} Z(t) dF(t) \end{aligned} \tag{28}$$

where $\bar{U}^S = \rho RV$ and

$$Z(t) = \begin{cases} [\theta + (1-\theta)K + (1-\theta)L - r_1\theta t - v_1(1-\theta)K]R & \text{if } 0 \le t \le t_0, \\ -r_2\theta(1-t) - D(1-\theta)L, & \\ 0, & \text{if } t_0 < t \le 1. \end{cases} \tag{29}$$

where $Z(t)$ denotes the equity owners' payoff from a bank that uses subordinated debt.

We can now consider the expected utility of a risk neutral agent. Using A.1 , (6), (7), (28) and (29) in (3), we get

$$EU^n = \rho RK + \frac{\rho}{\theta'}\{(1 - r_1 t^e)R - r_2(1 - t^e)\} + (1 - \rho R)v_1 K - \frac{EU^S - \overline{U}^S}{1 - \theta} \tag{30}$$

We will denote the solution in this section by using the superscript ** on the variable.

Proposition 3: *Let A.1 - A.3 and A.4′ hold. Assume that* $\underline{A} \le A \le \overline{A}$, $K < K_0(A)$ *and* $V \ge (1-\theta)(K_0(A) - K)$. *A Nash Equilibrium exists in which risk averse agents buy deposits and risk neutral agents buy equity,* $v_1^{**}(A) = v_1^*(A) = 0$ *and* $r_j^{**}(A) = r_j^*(A)$, $j = 1, 2$, *where* $r_j^*(A)$ *and* $v_1^*(A)$ *are as given in Proposition 1. In this equilibrium, agent S lends* $(1-\theta)L = (1-\theta)(K_0(A) - K)$ *to the bank in period* 0. *Also,*

$$0 < t_0^{**}(A) = \left[\frac{(2 - r_1^{**}(A))R - r_2^{**}(A) + 2\theta' RK}{r_1^{**}(A)R - r_2^{**}(A)}\right]^{(1/2)} < 1 \tag{31}$$

$$D^{**}(A) = R + \frac{(1 + \theta' K)R - t_0^{**}(A)(r_1^{**}(A)R - r_2^{**}(A)) - r_2^{**}(A)}{\theta'(K_0(A) - K)} > R \tag{32}$$

The bank is run-proof.

Proof: Please see the Appendix.

The intuition for $t_0^{**} < 1$ and $D^{**} > R$ is very simple. If $t_0^{**} < 1$, this means there will be a default on debt. If $D^{**} \le R$, then even when there is no default, the repayment on the loan is no greater than R. Thus, the expected return per unit of loan is less than R, which will mean the bank will not be able to raise debt capital. What happens if $K \geq K_0$? Rewrite (25) as

$$t_0 = \frac{(1+\theta' K)R - r_2}{r_1 R - r_2} + \frac{(R-D)\theta' L}{r_1 R - r_2}$$

If $K \geq K_0(A)$, we know that the first term on the right hand side is greater than equal to 1, since $r_1^* = r_1^{**} = 1 + \theta' K_0(A)$. So, debt capital becomes irrelevant to prevent the run and, if there is any debt in the bank, it will be risk free and, hence, D will then be equal to R. In the special case of $A = \overline{A}$, we have

$$t_0^{**}(\overline{A}) = \left[\frac{K}{K_0(\overline{A})}\right]^{\frac{1}{2}}$$

In section 2, we had shown that a bank with adequate equity capital finds it optimal to avoid bank runs and to allocate resources optimally. In this section, as in the previous section, we have shown that adequate *equity* capital is not necessary. Also, similar to that in Section 3, it is interesting that the present value of the debt must be no less than $(K_0(A) - K)$. The total of equity capital and debt capital must once again be equal to the capital required when the capital adequacy is met by equity only.

5 Conclusion

In this paper we treat the bank as a financial institution that offers demand deposits at fixed rates of interest. We first show that there exists a minimum amount of risk neutral equity capital, which will ensure a run-free bank. We then consider the situation where a bank does not have enough equity capital, but has access to a line of credit. We model the line of credit as a right to access funds at pre-determined rates of interest. However, for this facility, the bank has to pay an upfront fee in the initial period. We go on to show that this too can give us a run-free bank, provided the creditor has enough endowment to credibly offer the facility. We finally argue that the solution, which requires the least endowment, is one in which the credit facility collapses to an insurance scheme, with an upfront premium.

We then show that debt capital can be substituted for equity capital. However, to keep the bank run-free, the minimum sum of debt and equity capital required is the same as in the case where there is no debt.

All the three mechanisms, equity only, the line of credit (to augment equity), and debt and equity are modelled as market mechanisms requiring no government intervention. The basic insight is that, given the risk averse depositors' reservation utility, there is a minimum amount of risk neutral capital that a bank must have access to, to ensure a run-free bank. The nature of this capital is irrelevant and market mechanisms can provide the solution.

APPENDIX

In the Appendix, we will suppress the argument A from all variables, wherever it causes no confusion in the logic of the argument.

Proof of Proposition 1:

If λ is the Lagrange multiplier on C1 in optimization problem (P1), it is easy to show that $\lambda > 0$, i.e., $EU^a = A$. This, in turn, implies that

$$u'(r_1) = \rho R u'(r_2)$$

(a) We first show that at $r_1^*(A), r_2^*(A)$ and $v_1^* = 0$, there is no run if $K \geq K_0$. Observe that for there to be no run, we need $\underline{t} \geq 1$ and $r_1 R - r_2 > 0$.

$$r_1 R - r_2 = r_1 R - Q r_1 = (R - Q) r_1 > 0$$

since $R > Q \equiv (\rho R)^{(1/s)}$ and $\rho < 1$. From equation (5) in the text, we have

$$\begin{aligned} \underline{t} &= \frac{(1+\theta' K)R - v_1 \theta' K R - r_2}{R r_1 - r_2} \\ &= \frac{(1+\theta' K)R - r_2}{R r_1 - r_2} \\ &\geq 1 \end{aligned}$$

if $(1 + \theta' K) \geq r_1$, i.e.,

$$K \geq \frac{1}{\theta'}[r_1 - 1] \equiv K_0$$

(b) Since there is no run, $Z(.)$ in (4) is always non-negative and from (8) we know that $v_1^* = 0$. Solutions $r_1^*(A)$ and $r_2^*(A)$ are obtained from the discussions in the text relating to equations (8), (11) and (12). This also shows that $EU^a = A$.

(c) The bank will be formed only if $EU^n \geq \rho RK$. Since $A \leq \bar{A}$ and (from (10) and (11)) $r_1^*(A) \leq (2R)(2R)/(R+Q)$, and $v_1^* = 0$, we get $EU^n \geq \rho RK$.

(d) We need to show that in equilibrium, risk averse agents buy deposits and risk neutral agents buy equity. Consider risk averse agents first. If a risk averse agent buys equity, she gets $v_1^* = 0$ in period 1. Given A.3, it follows that she will not buy equity.

Next consider a risk neutral agent. If she deviates and invests in deposits, she gets

$$\begin{aligned} & t^e r_1^*(A)K + (1-t^e)\rho r_2^*(A)K \\ = & \frac{1}{2} r_1^*(A)K(1+\rho Q) \\ \leq & \frac{1}{2} r_1^*(\overline{A})K(1+\rho Q) \\ = & \frac{RK}{R+Q}(1+\rho Q) \\ < & \frac{RK}{R+Q}(\rho R+\rho Q) \\ = & \rho RK \end{aligned}$$

while EU^n is at least as much as ρRK. The first equality follows from the fact that $r_2^* = Qr_1^*$ and $t^e = (1/2)$ (from A.1). The first inequality follows from the fact that $r_1^*(A)$ is an increasing function of A till $\bar{A}$ (text immediately after (11)). The second equality follows from (10) and (11) that give us $r_1^*(\overline{A}) = \frac{2R}{R+Q}$. The strict inequality follows from A.2

Since no agent finds it optimal to deviate, the solution is a Nash equilibrium. ||

Proof of Proposition 2:

(a) For a no-run bank, we need $T \geq 1$ from (16). Using $\hat{v}_1 = 0$, and $\hat{r}_1 = r_1^* = 1 + \theta' K_0(A)$ from Proposition 1, $T \geq 1$ is equivalent to

$$[1+\theta' K]R + \theta' x[R(1-p) - r_C] \geq [1+\theta' K_0(A)]R$$

$$\Leftrightarrow \quad \theta' x[R(1-p) - r_C] \geq \theta'[K_0(A) - K]R$$

$$\Leftrightarrow \quad x \geq \frac{[K_0(A) - K]R}{R(1-p) - r_C}$$

which is true by hypothesis.

(b) Proof of $EU^a = A$ follows from Proposition 1 given the values of $\hat{r}_j$. To show $EU^C = \bar{U}^C$, consider the expression in braces on the right hand side of (19) which, using the value of p in this Proposition and A.1, can be written as

$$(R - r_C)(1 - \frac{t_C}{2}) - \frac{R - r_C}{t_C}\left[\frac{1}{2} - (\frac{1}{2} - t_C + \frac{{t_C}^2}{2})\right] = 0$$

Thus, $EU^C = \bar{U}^C$.

(c) From (21) and A.2, it is immediate that $\hat{v}_1 = 0$. Using the result in part (b), and the values of $\hat{r}_j$ $j = 1, 2$, we know that $EU^n \geq \rho RK$ (see Proposition 1). ||

Proof of Corollary 1:

Plugging in the values of $\hat{x}$, $\hat{p}$ and $\hat{r}_C$ in Proposition 3, it is immediate that the solution in the Proposition can be supported. We only need to prove the wealth requirement.

For this, first observe that,

$$(1-p)^2 = \left[\frac{\theta'}{r_1^*}\right]^2 \frac{(K_0(A) - K)r_1^*}{\theta'}$$

$$\Rightarrow \quad (1-p)^2 = \frac{(K_0(A) - K)\theta'}{2r_1^*}$$

$$\Rightarrow \quad (1-p)x = K_0(A) - K$$

From (17), it immediately follows that W must be greater than $\underline{W}$ for all conditions in Proposition 3 to be valid.

Using the lower bound of x in Proposition 2, we have

$$x \geq \frac{[K_0(A) - K]R}{R(1-p) - r_C}$$
$$\Rightarrow \quad (1-p)x \geq K_0(A) - K + \frac{r_C x}{R} > K_0(A) - K$$

if $r_C > 0$. ||

Proof of Proposition 3:

(a) We first show that there is no run on the bank. For this, we need $t_L \geq 1$ in (26). Observe that once we substitute $v_1^{**} = 0$, $L = K_0 - K$, and $r_j^{**} = r_j^*, \;\; j = 1, 2$, in (26), we can follow the exact same steps as in part (a) of the proof to Proposition 1 to argue that there is no run on this bank.

(b) Following the same method as in the proof to Proposition 1, we have $EU^a = A$. Here we also need to show that $EU^S = \bar{U}^S$. Suppose instead, that $EU^S > \bar{U}^S$. Observe that if we reduce D in (29), the value of $Z(t)$ increases. Then, we do not affect (26) and any other equilibrium value, but we increase EU^n (since Z increases) and reduce EU^S (see (28)). Hence, in equilibrium, $EU^S = \bar{U}^S$.

(c) From equation (30), it is immediate that $v_1^{**} = 0$. Given part (b) and $r_j^{**} = r_j^*, \;\; j = 1, 2$, we know that (as in part (c) of Proposition 1) $EU^n \geq \rho RK$.

(d) The fact that risk-averse buy deposits and risk-neutral buy equity in the bank, follows the same steps as in part (d) of Proposition 1, given our equilibrium values of r_j^{**} and v_1^{**}.

(e) That leaves us with calculating the values of D^{**} and t_0^{**}. From (29),

$$\int_0^1 Z(t)dF(t) \quad = \quad \int_0^{t_0} \Big\{ [\theta + (1-\theta)K + (1-\theta)L - r_1\theta t - v_1(1-\theta)K]R$$

$$
\begin{aligned}
& -r_2\theta(1-t) - D(1-\theta)L\Big\}dF(t) \\
= \;& \theta\Big\{[1+\theta' K(1-v_1)]R - r_2 + (R-D)\theta' L\Big\}t_0 \\
& -(r_1R - r_2)\theta\frac{t_0^2}{2} \\
= \;& (r_1R - r_2)\theta\frac{t_0^2}{2}
\end{aligned}
$$

where the last equality follows after using (25). Using the last expression for $\int_0^1 Z(t)dF(t)$ in (28), we get

$$
\begin{aligned}
EU^S \;=\; & \overline{U}^S + \rho\Big\{-(r_1R - r_2)\theta\frac{t_0^2}{2} + \theta\{(1-r_1t^e)R - r_2\theta(1-t^e)\} \\
& +(1-\theta)KR - v_1(1-\theta)KR\Big\}
\end{aligned}
$$

In equilibrium, $EU^S = \overline{U}^S$, $r_1 = r_1^{**}(A)$, $r_2 = r_2^{**}(A)$ and $v_1 = v_1^{**} = 0$. Hence,

$$
-(r_1^{**}(A)R - r_2^{**}(A))\theta\frac{t_0^2}{2} + \theta\{(1-r_1^{**}(A)t^e)R - r_2^{**}(A)(1-t^e)\} + (1-\theta)KR = 0
$$

Using A.1, we get the equilibrium value of t_0 as given in (31). Taking $t_0 = t_0^{**}(A)$ in (25), we get $D^{**}(A)$ (as in (32)).

To show $0 < t_0^{**} < 1$, it is sufficient to show that $0 < [t_0^{**}]^{(1/2)} < 1$. From (31),

$$
\begin{aligned}
& \frac{(2-r_1^{**})R - r_2^{**} + 2\theta' RK}{r_1^{**}R - r_2^{**}} \\
= \;& \frac{2(1+\theta' K)R - (R+Q)r_1^{**}}{r_1^{**}(R-Q)} \\
> \;& \frac{2R + 2\theta' KR - 2R}{r_1^{**}(R-Q)} \\
= \;& \frac{2\theta' KR}{r_1^{**}(R-Q)} > 0
\end{aligned}
$$

The first equality follows from $r_2^{**} = r_2^* = Qr_1^* = Qr_1^{**}$ (Proposition 1); the first inequality follows from the discussions in the text around equations (8) to (11). To show that $t_0^{**} < 1$, suppose, on the contrary that $t_0^{**} \geq 1$. Then, from (31),

$$
\begin{aligned}
& 2(1+\theta' K)R - (R+Q)r_1^{**} \geq r_1^{**}(R-Q) \\
\Rightarrow \quad & 2R + 2\theta' KR \geq r_1^{**} 2R \\
\Rightarrow \quad & 1+\theta' K \geq r_1^{**} = r_1^{*} = 1+\theta' K_0
\end{aligned}
$$

which is not possible since by assumption $K < K_0$.

We now show that $D^{**} > R$. Suppose, instead, that $D^{**} \leq R$. Since $t_0^{**} < 1$, we can rewrite (27) as

$$
\begin{aligned}
EU^S &< \rho\{[V-(1-\theta)L]R + D(1-\theta)L\} \\
&= \rho VR + \rho(D-R)(1-\theta)L \leq \rho VR
\end{aligned}
$$

which is not possible. ||

References

[1] Akerlof, George A., 1970, The market for "lemons": Qualitative uncertainty and the market mechanism, *Quarterly Journal of Economics* 84, 488-500.

[2] Allen, Franklin, and Douglas Gale, 1998, Optimal financial crises, *Journal of Finance*, 53(4), 1245-84.

[3] Bhattacharya, S., and A. Thakor, 1993, Contemporary banking theory, *Journal of Financial Intermediation* 3, 2-50.

[4] Bhattacharya, S., A. Boot, and A. Thakor, 1998, The Economics of Bank Regulation, *Journal of Money, Credit and Banking* 30, 745-70.

[5] Bryant, J., 1980, A model of reserves, bank runs, and deposit insurance, *Journal of Banking and Finance* 4, 335-44.

[6] Buser, S. A., A. Chen and E.J. Kane, 1981, Federal deposit insurance, regulatory policy, and optimal bank capital, *The Journal of Finance* 35, 51-60.

[7] Calomiris, Charles W., and Gary Gorton, 1991, The origins of banking panics: Models, facts, and bank regulation, in R. Glen Hubbard, ed: *Financial Markets and Financial Crises* (University of Chicago Press, Chicago).

[8] Chan, Y.S., S.I. Greenbaum, and A.V. Thakor, 1992, Is fairly priced deposit insurance possible? *The Journal of Finance* 47, 227-45.

[9] Demirgüç-Kunt, A., and E. Detragiache, 1997, The determinants of banking crises: Evidence from developed and developing Countries, *mimeo* (The World Bank).

[10] Diamond, Douglas W., and Philip H. Dybvig, 1983, Bank runs, deposit insurance and liquidity, *Journal of Political Economy* 91, 401-19.

[11] Diamond, Douglas W., and Raghuram G. Rajan, 2000, A theory of bank capital, *The Journal of Finance* 55, 2431-66.

[12] Diamond, Douglas W., and Raghuram G. Rajan, 2001, Liquidity risk, liquidity creation and financial fragility: A theory of banking, *Journal of Political Economy* 109, forthcoming.

[13] Freixas, Xavier, and Jean-Charles Rochet, 1997, *Microeconomics of Banking* (MIT Press, Cambridge, USA).

[14] Gangopadhyay, Shubhashis, and Gurbachan Singh, 2000, Avoiding bank runs in transition economies: The role of risk neutral capital, *Journal of Banking and Finance* 24, 625-42.

[15] Gangopadhyay, Shubhashis, and Gurbachan Singh, March, 2001, Capital Adequacy for Run-proof Banking: A market alternative to deposit insurance and lender of last resort *Indian Statistical Institute, Delhi Centre Discussion Paper 01-04.*

[16] Gorton, Gary, and Andrew Wilson, 2000, Liquidity provision, bank capital, and the macroeconomy, *University of Minnesota, Department of Finance Working Paper 9806.*

[17] Haubrich, Joseph J., and Robert G. King, 1990, Banking and insurance, *Journal of Monetary Economics* 26, 361-86.

[18] Jacklin, C. J., 1987, Demand deposits, trading restrictions, and risk sharing, in Edward D. Prescott and Neil Wallace, eds: *Contractual Arrangements for Intertemporal Trade* (University of Minnesota Press, Minneapolis).

[19] Kane, Edward J., 1985 *The Gathering Crisis in Deposit Insurance* (MIT Press, Cambridge, USA).

[20] McCulloch, J.H., and M. Yu, 1998, Government deposit insurance and the Diamond-Dybvig model, *Geneva Papers on Risk and Insurance Theory* 23(2), 139-49.

[21] Selgin, George, 1996, *Bank Deregulation and Monetary Order* (Routledge, London and New York).

[22] Sharpe, W. F., 1978, Bank capital adequacy, deposit insurance, and security values, *Journal of Financial and Quantitative Analysis: Proceedings Issue* 13, 701-18.

[23] Singh, Gurbachan, 2003, *Some aspects of banks and financial markets in emerging economies* Ph.D. dissertation, Indian Statistical Institute, Delhi.

[24] Sleet, Christopher, and Bruce D. Smith, 2000, Deposit Insurance and Lender-of-Last-Resort Functions, *Journal of Money, Credit and Banking* 32, 518-75.

[25] Stevens, Ed, 2000, Comment on deposit insurance and lender-of-last-resort functions, *Journal of Money, Credit and Banking* 32, 576-79.

	Risk averse	Risk neutral	Total
Type 1	θt	$(1-\theta)t$	t
Type 2	$\theta(1-t)$	$(1-\theta)(1-t)$	$(1-t)$
Total	θ	$(1-\theta)$	1

Table I: Distribution of agents by risk aversion and by utility function